CALIBRATING a BIFILAR MICROMETER by MEASURING DECLINATION DIFFERENCES of H.I.C. STARS in the PLEIADES

ABSTRACT

One of the methods suggested in the Double Star Observer's Handbook [1] for calibrating the screw constant of a filar micrometer, is by measuring declination differences of stars in the Pleiades, following the recommend-ation of Aitken [2]. Because Aitken's book 'The Binary Stars' is very difficult to obtain I have prepared a short paper describing the method in detail, and how it has been used to calibrate my Grubb bifilar micrometer.

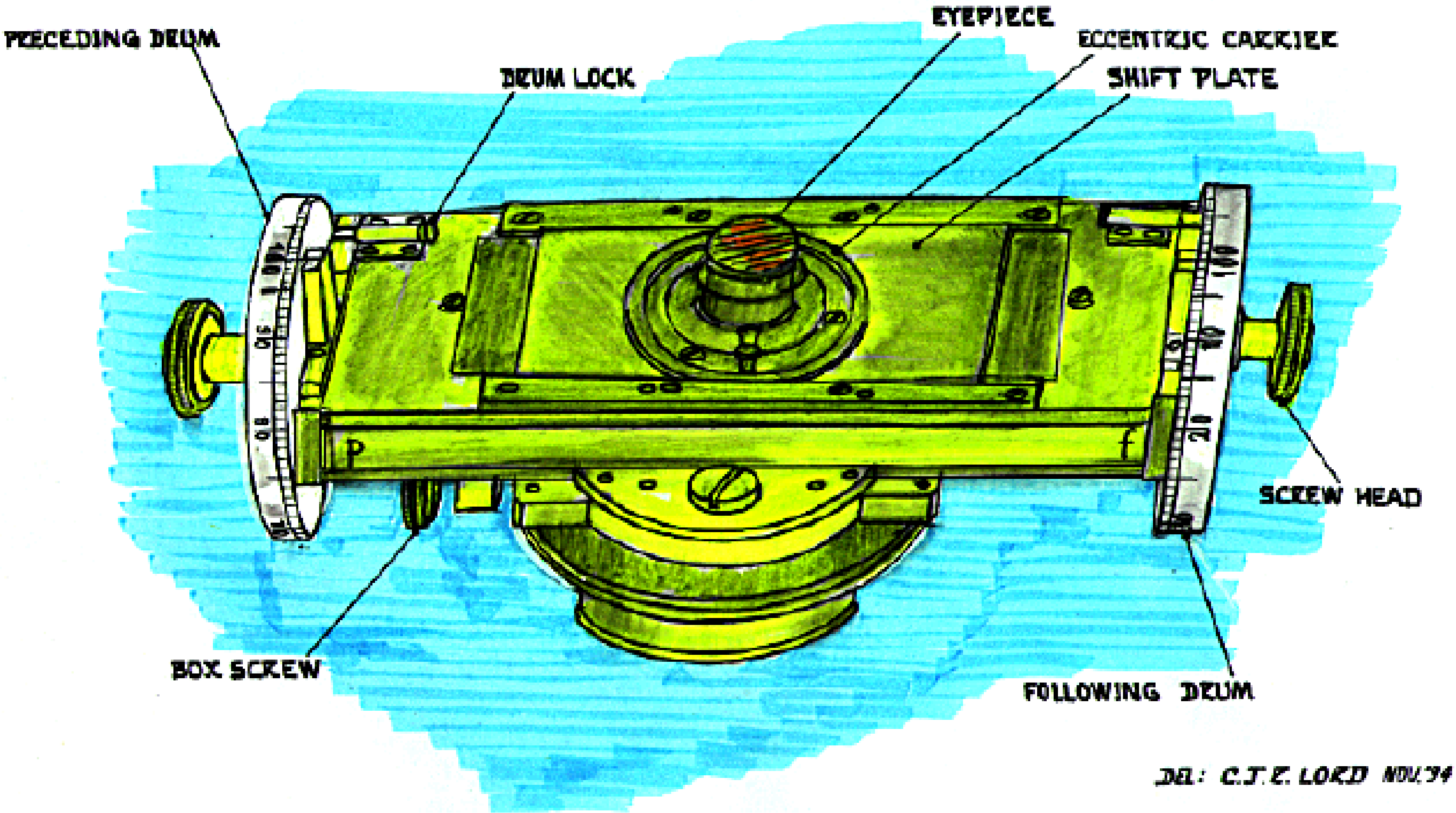
INTRODUCTION

The most common method of calibrating the screw constant of a filar micrometer is by measuring relfix pairs of wide separation. This method can be extended to the measurement of declination differences provided the screw has adequate travel and the eyepiece can be moved between the fixed and travelling webs. The wider the arc that may be spanned the greater the calibrating accuracy because measurement error is independent of fiducial distance, and the screw constant may be averaged over the number of revolutions needed to encompass the separation. Measurement of declination differences of pairs having similar RA's and magnitudes reduces the effects of drive errors and ensures the highest calibration accuracy. Stars within the Pleiades are ideally suited to this purpose.

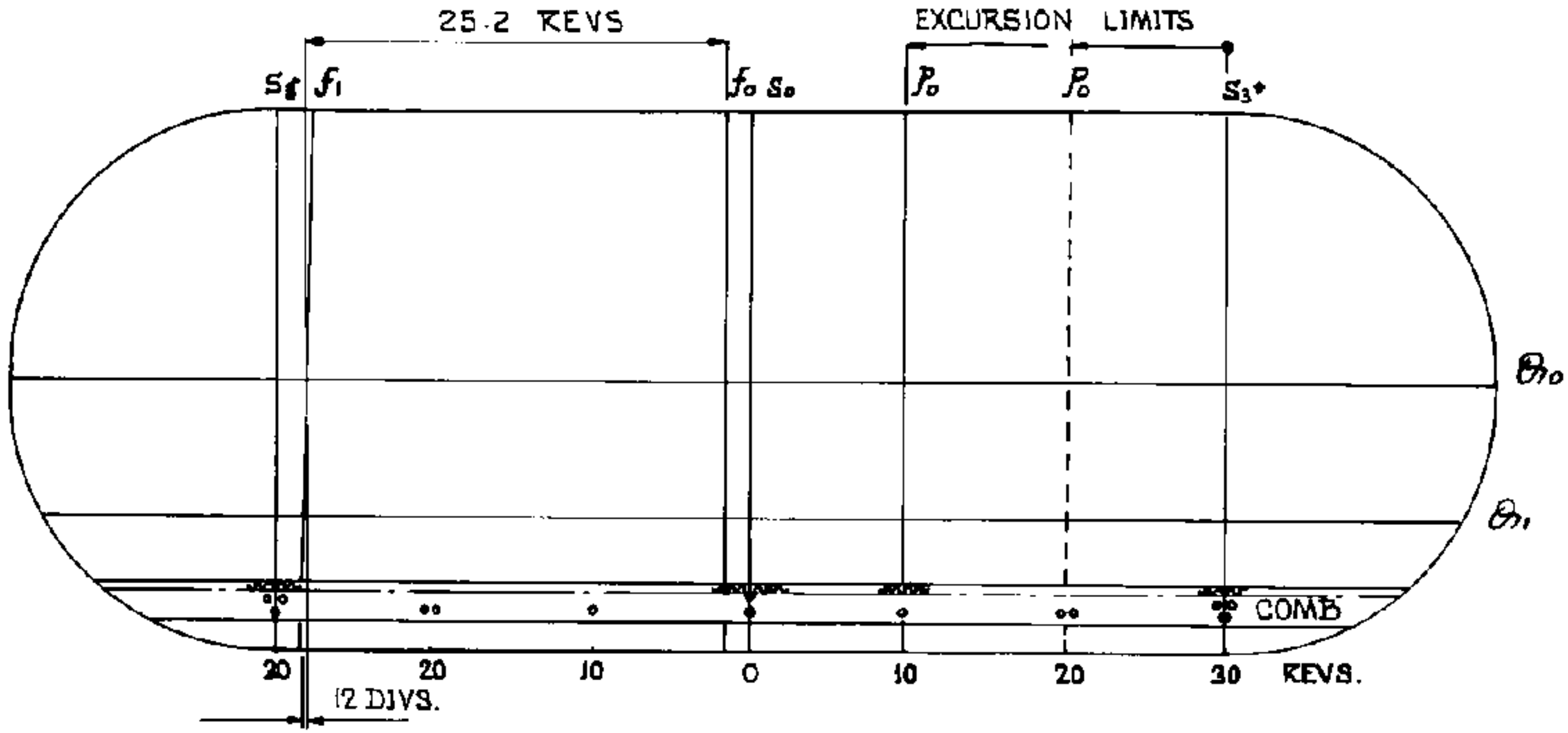
In August 1995 I was presented with a fine bifilar micrometer manufactured for use with a 15inch $f/12$ refractor by Grubb, in 1895. I rewebbed it using 10μ quartz filaments and fitted 3000mCd clear encapsulated red LEDs to illuminate the webs and a stepped mains transformer and wire wound potentiometer to vary the brightness. The 50 TPI screws are furnished with $2\frac{1}{2}$ INCH diameter drums divided into 100 divisions, and have over 80 revolutions travel. It was more than 12 years since I rewebbed a micrometer and I found the task extremely taxing. Having tried both 8μ wire and spider's web in the past as well as 10μ quartz filaments, I definitely prefer the latter. They are delicate, yet easier to handle and 'super-glue' in place. What made the task awkward was the design of the frames. Instead of being rectangular they were open ended, like tuning forks, and their sides sprang apart slightly when they were removed from their guides within the micrometer housing. When replaced the webs sagged so the frames had to be webbed in situ. One inadvertent twitch of the hand and the quartz filaments already glued in placed were broken. Eventually after two eight hour sessions the job was satisfactorily completed and I breathed an immense sigh of relief directed well away from the micrometer! I later modified the web arrangement and shimmed the frames to ensure the separation webs and position webs just cleared, yet remained within the depth of focus.

The 10μ quartz filaments were obtained from the ATM, J.D. Greenwood [3]. The London instrument maker Irving [4] manufactured brass adaptors in the style of Grubb to couple the bifilar micrometer to a $4\frac{1}{2}$ INCH Position Circle and my 6 INCH Maksutov-Cassegrain.

GRUBB BIFILAR MICROMETER CIRCA 1895



GRUBB BIFILAR MICROMETER WEB CONFIGURATION



$$R_o = \frac{648.10^3}{\pi} \cdot P_o \frac{(1 + 7.10^{-6}(t_o - t))}{F_o}$$

$$P_o = P(1 + 7.10^{-6}(t_o - 68))$$

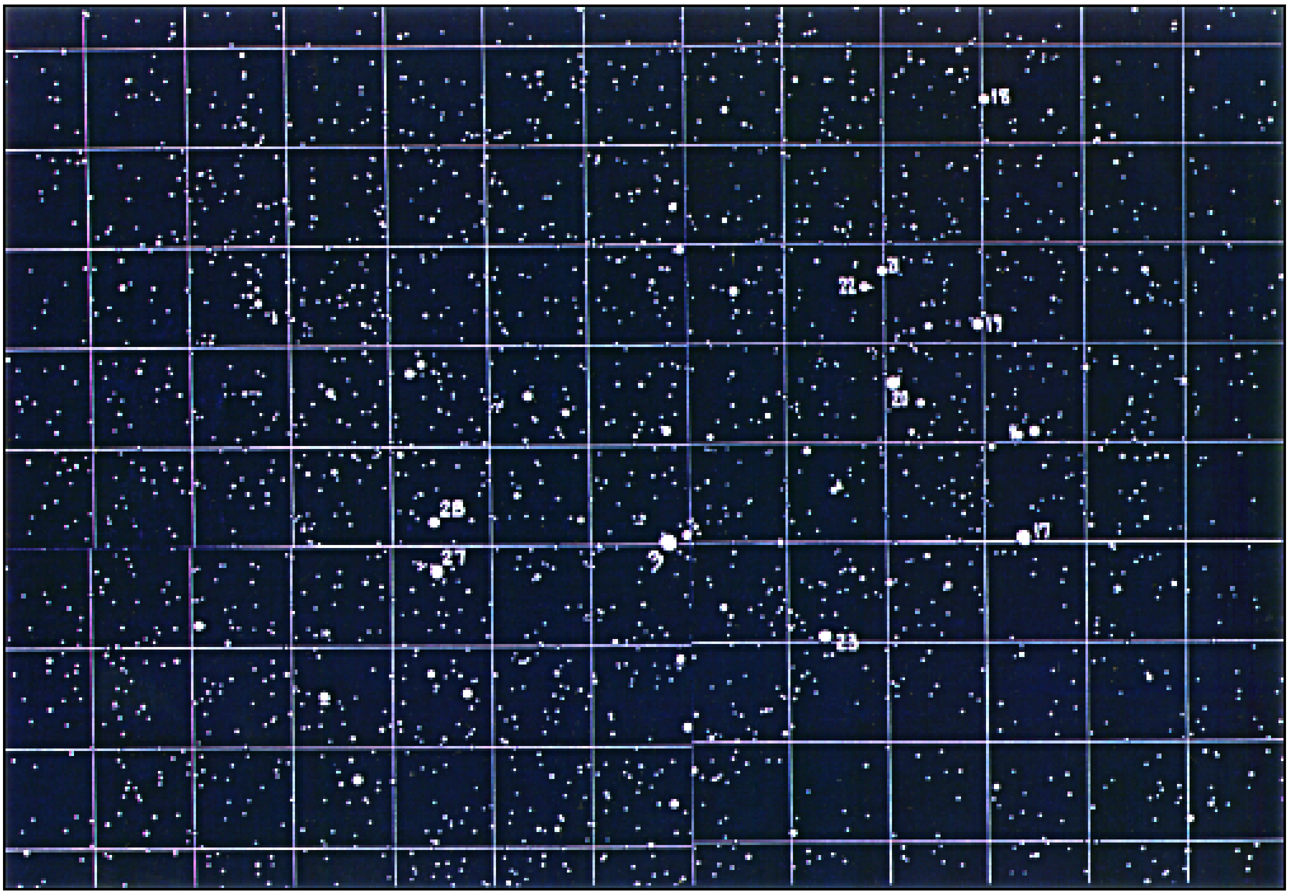
$$t_o = 32°F$$

To help determine the RA's & DEC's of some of the naked eye stars in the Pleiades, Bob Argyle [5] kindly provided the following table:

Star	HIC	RA (J2000)	DEC (J2000)	PMRA	PMDEC
				secs	arcsecs
16Tau	17489	3 44 48.184	24 17 21.33	0.010	-0.054
17Tau	17499	3 44 52.532	24 06 48.01	0.019	-0.046
18Tau	17527	3 45 09.714	24 50 21.96	0.013	-0.038
19Tau	17531	3 45 12.55	24 18 01.8	0.042	-0.048
20Tau	17573	3 45 49.565	24 22 03.35	0.007	-0.058
21Tau	17579	3 45 54.414	24 33 15.96	0.004	-0.049
22Tau	17588	3 46 02.869	24 31 40.11	0.010	-0.048
23Tau	17608	3 46 19.546	23 56 53.31	0.014	-0.061
25Tau	17702	3 47 29.073	24 06 18.38	0.019	-0.046
27Tau	17847	3 49 09.739	24 03 12.24	0.018	-0.047
28Tau	17851	3 49 11.190	24 08 12.47	0.015	-0.047

The J2000 positions are accurate to the number of decimal places shown and are extracted from the Hipparcos Input Catalogue.

The chart accompanying it is abstracted from Oort's 'Atlas of the Pleiades' edited by Ejnar Hertzsprung [6].



DETERMINATION of DECLINATION DIFFERENCE

The positions have to be precessed from J2000 to the equinox of date and corrected for proper motion and differential refraction. To do this the following reduction formulae are used:

FOR REDUCTION FROM J2000 TO EQUINOX OF DATE

$$\alpha = \alpha_0 + M + N \sin\alpha_m . \tan\delta_m$$

$$\delta = \delta_0 + N \cos\alpha_m$$

$$\alpha_m = \alpha_0 + \tfrac{1}{2}\left(M + N \sin\alpha_0 . \tan\delta_0\right)$$

$$\delta_m = \delta_0 + \tfrac{1}{2} N \cos\alpha_m$$

$$M = 1^{\circ}.281\,2323T + 0^{\circ}.000\,3879T^2 + 0^{\circ}.000\,0101T^3$$

$$N = 0^{\circ}.556\,7530T - 0^{\circ}.000\,1185T^2 - 0^{\circ}.000\,0116T^3$$

$$T = (t - 2000)/100$$

FOR DETERMINATION OF DIFFERENTIAL REFRACTION

$$\Delta R = 60''.4\left(\tan z_1 - \tan z_2\right)$$

$$\cos z = \sin\delta . \sin\phi + \cos\delta . \cos\phi . \cos HA$$

$$HA = GST - \lambda - RA$$

APPARENT POSITION

$$\alpha' = \alpha - (t - 2000)\mu_\alpha + \Delta r$$

$$\delta' = \delta - (t - 2000)\mu_\delta + \Delta r$$

The table below provides 1995.0 positions of the principal naked eye stars corrected for proper motion, the declination differences, and the apparent differences on the meridian from London.

M45 HIPPARCOS INPUT CATALOGUE STARS

HIC	TAU	RA (1995)	DEC (1995)
17489	16CELAENO	3 44 30.539	24 16 25.70
17499	17ELECTRA	3 44 34.579	24 05 52.46
17527	18	3 44 51.119	24 49 26.48
17531	19TAYGETA	3 44 54.44	24 27 06.4
17573	20MAIA	3 45 31.635	24 21 08.21
17579	21ASTEROPE	3 45 36.630	24 32 20.78
17588	22ASTEROPE	3 45 45.761	24 30 45.00
17608	23MEROPE	3 46 01.628	23 55 58.32
17702	25ALCYONE	3 47 11.102	24 05 23.78
17847	27ATLAS	3 48 51.767	24 02 18.27
17851	28PLEIONE	3 48 53.223	24 07 18.51

TAU	DEC.DIFFS.	DIFF.REF.	APP.DEC.DIIFS.
17-16	10' 33".24	-0".24	10' 33".00
18-19	22' 20".10	-0".50	22' 19".60
19-16	10' 40".68	-0".23	10' 40".45
19-17	21' 13".92	-0".47	21' 13".45
21-20	11' 12".57	-0".25	11' 12".32
28-27	5' 00".24	-0".11	5' 00".13

MAKING THE SEPARATION MEASURES

Each pair was measured, first with the p screw fixed, in both directions, the micrometer reversed and the procedure repeated with the *f* screw fixed. Each measure was repeated on six separate nights, the Pleiades within an hour of the meridian. It helps if the calibration measures are binned in roughly 5’; 10’; & 20’ arc, for both screws, enabling pitch variations to be investigated.

REDUCING THE CALIBRATION MEASURES

It helps in making the reduction if the web configuration has been previously calibrated. The manufacturer provides ruled scribe lines on the frames in which the webs may be placed. However the webs may fall anywhere within them and their separations must be measured, and the parallelism error of the p & *f* webs measured also. The accompanying figure depicts the web configuration of my Grubb bifilar micrometer. Each frame carries the separation webs f_0, f_1 & p_0; the eyepiece plate carries two parallel position webs ϑ_0& ϑ_1 and three fixed separation webs, s_0, s_{3+}, s_{3-}. Web p_0 has a restricted excursion. Webs f_0 & f_1 are separated by 25.2 revolutions. The screw drums are clutched, permitting the comb to be zeroed for the p_0 & f_0 webs.

Backlash in micrometers manufactured by Grubb and other contemporary instrument makers may be adjusted. Each frame is connected to its screw by a spring loaded bush. However the greater the pressure on the screw the harder it becomes to turn and the bush wears more rapidly. I adjusted the bushes until backlash amounted to 40 drum divisions for each screw.

I collected the sets of measures into three bins, those at approximately 20’arc; 10’arc & 5’arc. Because more than thirty measures were made for 20’arc bin the error of the mean was determined from the sample standard deviation; for the 10’arc & 5’arc bins the population standard deviation was used.

REDUCTION *of* MEASURES

The frequency distribution of unbiased measures will be approximately normal. Measures are reduced in groups binned into apparent declination differences:

Tau	1995 SEP	REVOLTIONS	WEIGHT
18-19	1339".60	38	4
19-17	1273".45		
21-20	672".32		
19-16	640".45	19	2
17-16	633".0		
28-27	300".24	9	1

The sample (n>30) or population (n<30) standard deviation is determined for each pair and the probable error determined from the 50% confidence limit.

In obtaining the mean of the screw constants within each group the range is determined from the probable error fiducial limits.

In determining the mean of all three groups, each group is weighted according to the ratio of the number of screw revolutions

The error is essentially due to the arc subtended by the 10µ webs. The calibration error is the combination of the measurement error and the position error. Both are negligible and quite inconsequential over such a wide arc which is the virtue of this method if you possess the patience to follow it through.

The screw constant (for each screw) is determined by dividing the mean apparent separation by the mean screw travel. If the pitch errors of the screw are small and random, and if the bushes are not worn, the accuracy of the calibration will be directly proportional to the screw travel.

The apparent declination difference is given by:

$$\delta_1 - \delta_2 = \frac{648.10^3}{\pi} \cdot \frac{l}{F}$$

where l = screw travel
F= effective focal length

& the screw constant by:

$$R = (\delta_1 - \delta_2) . \frac{p}{l}$$

where p =screw pitch

hence, by substitution & eliminating F:

$$R = (\delta_{\bar{1}} - \delta_{\bar{2}}) . \frac{p}{l}$$

CALIBRATION ACCURACY

Error in calibrating the screw constant is given by:

$$\Delta R = -\frac{\Delta l}{l^2} . p(\delta_{\bar{1}} - \delta_{\bar{2}})$$

the value of Δl is derived from the probable error (50% confidence limit) of the standard deviation of the mean separation measures,

hence: $\Delta l = 0.6745\sigma_{n-1} l$

Errors in calibration add to errors in measurement. Providing the calibration error is at least an order of magnitude smaller than the measurement error, it is not significant; it cannot effect the error calculated from the standard deviation of the mean of the separation measures. The limiting measurement accuracy is governed by the size of the point spread function (Airy disc). the subtended arc of the webs, and the repeatability of the convergence of the separation webs in establishing the zero point, which except for pairs at or close to the Rayleigh limit, is independent of fiducial distance.

Repeatability of separation web convergence may be estimated by a bench determination of the zero error. A typical value would correspond to half the web thickness. The determination should be made for both increasing and decreasing drum readings, averaged from more than 30 estimates. The mean difference is a measure of the screw backlash.

It must be born in mind that the repeatability error may be additive or subtractive. Of a mean of 37 decreasing f drum readings the zero error was found to be $99.081^{\pm 0.882}$ and of 38 increasing readings $39.158^{\pm 0.819}$. The errors are marginally less than the web radius (linear dia. of web = 10µ = 0.4thou = 2divs.)

For the subtended arc of the web not to exceed the diameter of the point spread function central light, the $f/no. \geq 1490\omega_{mm}$. For 10µweb, the limiting focal ratio is $f/15$. If the effective focal ratio is $f/30$ the subtended arc of the web will be half the diameter of the PSF central light and so on.

The minimum calibration arc required to provide a calibration error ten times less than the measurement error:

putting $\Delta\rho \approx \phi_\omega$ where ϕ_ω is the subtended arc

$$\Delta R \leq 0.1\phi_\omega$$

for

$$\Delta R = \phi_\omega . \frac{p}{l} = 0.1\phi_\omega$$

hence $l_{\min} = 10p = 10$ revolutions of the micrometer screw

The declination differences measured encompassed approximately 10, 20 & 40 revolutions of a 50TPI pitch screw. The calibration error amounted to only ±0".07arc. The diameter of the PSF central light (D = 6 INCHES) is 0".91arc, and the angle subtended by the 10µweb is 0".68arc.

If the web subtends an arc smaller than the PSF central light, and provided the eyepiece can be moved along the position web so each star may be centred in the F.O.V., measurement errors can be achieved of the order of the subtended web arc.

Because the effective focal length and screw pitch vary with temperature, the calibrated screw constant may be usefully checked by measuring a wide relfix pair at the start and end of each session. The wider the separation the better, at least 10 revolutions.

The calibrated screw constant may be reduced to $R_0 @ t_0 = 32^\circ F$ using the expression:

$$R_0 = \frac{648.10^3}{\pi} . \frac{p_0\left(1 + 7.10^{-6}(t_0 - t)\right)}{F_0}$$

This method was advocated by Aitken [2]. However the variation of F with temperature in compound telescopes can be much more significant than the contraction/expansion of the steel screw. Unless you intend to measure very wide pairs, $\rho \geq 100''$arc, the difference is negligible. The pitch at the zero point is given by:

$$p_0 = p\left(1 + 7.10^{-6}\left(t_0 - 68\right)\right)$$

It must be born in mind that reducing the calibrated screw constant to R_0 involves two operations; reducing the pitch p_0 and then reducing the calibration to R_0. The difference may seem trivial but if the screw constant is determined (and quoted) to three decimal places it will have an effect.

REFERENCES

1. Webb Society Deep Sky Observer's Handbook - Vol.1 Double Stars, 2nd. Ed. p30 Enslow Publishers Inc.
2. Aitken, R.G. 'The Binary Stars' - Dover 3rd. Ed. 1965.
3. Greenwood, J.D., Supplier of 10m quartz filaments, 1, Westgate Avenue, Morecambe, Lancashire, LA3 3DW, UK.
4. Irving, H.N. & Son, Instrument Maker, 258, Kingston Road, Teddington, Middlesex, LONDON, TW11 9JQ, UK.
5. Argyle, R.W., M.Sc.,F.R.A.S. R.G.O. Cambridge, UK.
6. Oort, Prof. Dr. J.H., Catalogue de 3259 Etoiles dans les Pléiades, discutés par Ejnar Hertzsprung - Annales Van de Sterrewacht te Leiden, Deel XIX, 1947