NEWTONIAN DIAGONAL - Minor Axis & Offset

DERIVATION of EQUATIONS

Minor Axis: $a = y_a - y_b$

Ofset: $b = x_c - d = \frac{1}{2}(x_a - x_b) - d$

Equations of condition:

$$y_1 = m_1.x_1 + c_1; \quad c_1 = \frac{h}{2}; \quad m_1 = \frac{(D-h)}{2(F-s)} = P$$

$$y_2 = m_2.x_2 + c_2; \quad c_2 = d; \quad m_2 = -1$$

$$y_3 = m_3.x_3 + c_3; \quad c_3 = \frac{-h}{2}; \quad m_3 = \frac{-(D-h)}{2(F-s)} = -P$$

$at \quad (x_a, y_a) \quad y_1 = y_2$

$$\therefore m_1.x_a + c_1 = m_2.x_a + c_2$$

$$\therefore x_a = \frac{c_2 - c_1}{m_1 - m_2} = \frac{d - \frac{h}{2}}{P+1}$$

$$\& \quad y_a = m_2.x_a + c_2 = \frac{d + \frac{h}{2}}{P+1}$$

$at \quad (x_b, y_b) \quad y_2 = y_3$

$$\therefore x_b = \frac{c_3 - c_2}{m_3 - m_2} = \frac{d + \frac{h}{2}}{1-P}$$

$$\& \quad y_b = m_2.x_b + c_2 = \frac{d - \left(d + \frac{h}{2}\right)}{1-P}$$

Minor Axis: $a = y_a - y_b = \dfrac{h + 2Pd}{1 - P^2}$

Offset: $b = \frac{1}{2}\left(x_a + x_b\right) - d = \dfrac{d + P.\dfrac{h}{2}}{1 - P^2} - d$

TRANSFORMATIONS

Minor Axis:

$$a = y_a - y_b = \frac{d - \left(d - \frac{h}{2}\right)}{1 + P} - \frac{d - \left(d + \frac{h}{2}\right)}{1 - P}$$

$$\therefore a = \frac{2(F-s)\left(d + \frac{h}{2}\right)}{2(F-s) - (D-h)} - \frac{2(F-s)\left(d - \frac{h}{2}\right)}{2(F-s) + (D-h)}$$

put

$$M = 2(F-s) - (D-h)$$

$$N = 2(F-s) + (D-h)$$

then

$$a = \frac{h(F-s) + 2d(F-s)}{M} - \frac{2d(F-s) - h(F-s)}{N}$$

$$= \frac{N\left(h(F-s) + 2d(F-s)\right) + M\left(h(F-s) - 2d(F-s)\right)}{MN}$$

$$= \frac{\left(2(F-s) + (D-h)\right)\left(h(F-s)\right) + \left(2(F-s) - (D-h)\right)\left(h(F-s) - 2d(F-s)\right)}{MN}$$

$$= \frac{(F-s)\left(2h(F-s) + h(D-h) + 4d(F-s) + 2d(D-h)\right) + (F-s)\left(2h(F-s) - h(D-h) - 4d(F-s) + 2d(D-h)\right)}{MN}$$

$$= \frac{(F-s)\left(4h(F-s) + d(D-h)\right)}{MN}$$

$$= \frac{4(F-s)\left(h(F-s) + d(D-h)\right)}{MN}$$

put

$$L = h(F - s) + d(D - h)$$

$$\therefore a = \frac{4(F - s).L}{MN}$$

but

$$M + N = 4(F - s)$$

$$\therefore a = \frac{(M + N).L}{MN} = \frac{L}{M} + \frac{L}{N}$$

TRANSFORMATIONS

Offset:

$$b = \tfrac{1}{2}\left(x_a + x_b\right) - d = \frac{d - \frac{h}{2}}{2(1+P)} + \frac{d + \frac{h}{2}}{2(1-P)} - d$$

$$\therefore b = \frac{(F-s)\left(d - \frac{h}{2}\right)}{2(F-s)+(D-h)} + \frac{(F-s)\left(d + \frac{h}{2}\right)}{2(F-s)-(D-h)} - d$$

$$= \frac{M\left(F - s\left(d - \frac{h}{2}\right) + N(F-s)\left(d + \frac{h}{2}\right) - MNd\right)}{MN}$$

$$= \frac{\left(2(F-s)-(D-h)\right)\left((F-s)\left(d - \frac{h}{2}\right)\right) + \left(2(F-s)+(D-h)\right)\left((F-s)\left(d + \frac{h}{2}\right)\right) - \left(2(F-s)-(D-h)\right)\left(2(F-s)+(D-h)\right).d}{MN}$$

$$= \frac{(F-s)\left(2d(F-s) - h(F-s) - (D-h)\left(d - \frac{h}{2}\right)\right) + (F-s)\left(2d(F-s) + h(F-s) + (D-h)\left(d + \frac{h}{2}\right)\right) - d\left(4(F-s)^2 + (D-h)^2\right)}{MN}$$

$$= \frac{(F-s)\left(h(D-h) + 4d(F-s)\right) - \left(4d(F-s)^2 + d(D-h)^2\right)}{MN}$$

$$= \frac{(D-h)\left(h(F-s) + d(D-h)\right)}{MN}$$

but

$$N - M = 2(D-h)$$

$$\therefore b = \frac{(N-M).L}{2MN} = \frac{1}{2}\left(\frac{L}{M} - \frac{L}{N}\right)$$