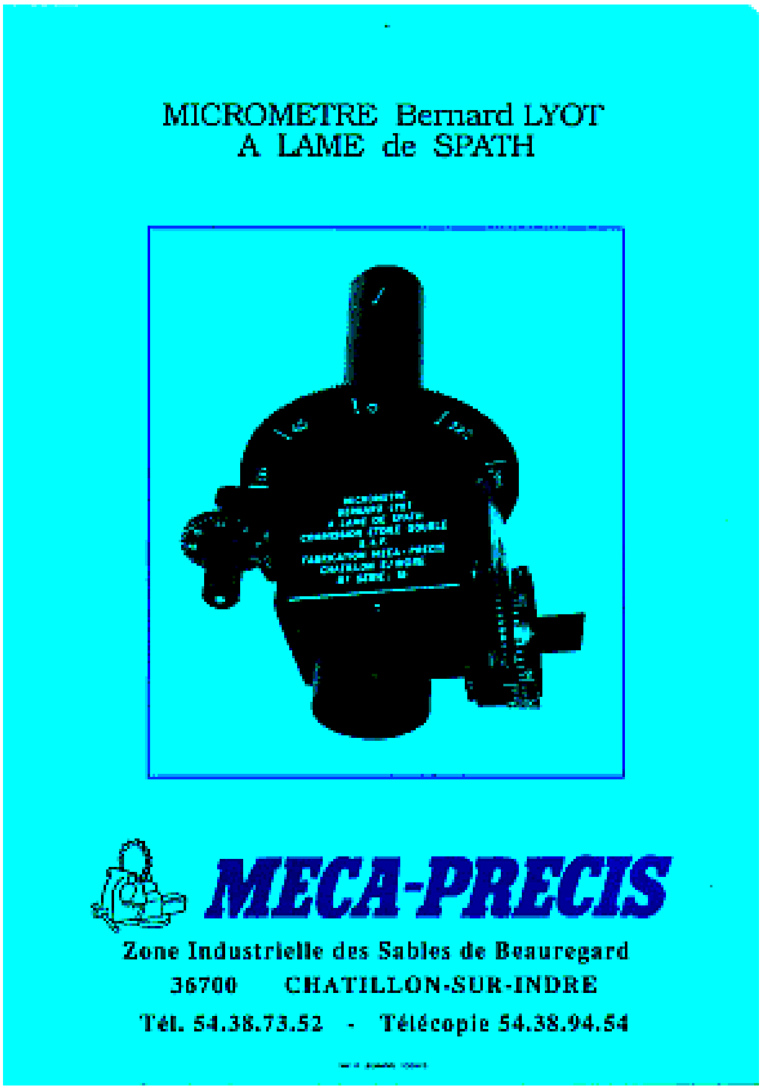


MEASURING CLOSE DOUBLE STARS WITH A LYOT MICROMETER

Abstract: The author describes the use of the Meca-Precis Lyot double image micrometer, designed by Jean-Louis Agati & Rene-Georges Huret, on behalf of the double star commission of the Societe Astronomique deFrance. A description of the micrometer, its optical principles and performance characteristics and the equivalent characteristics of the filar micrometer are discussed.

Introduction: Historically the preferred method of measurement of PA & Sep. has been by wire micrometer. The mechanical principles upon which a filar micrometer works are easily understood, and the necessary technique a matter of practice. The accessibility of the technology has tended to outweigh the practical difficulties in making a measurement, difficulties which lead to significant errors near the Rayleigh limit.

Less well known is the Lyot Spath Blade micrometer, invented by Bernard Lyot in 1949 and first used by Henri Camichel. The principles upon which the micrometer works are less obvious than other image micrometers, such as the Wollaston and the Muller prism, or the diffraction micrometer. The principal advantages lie in its compactness, its inherent sensitivity near the Rayleigh limit, and its insensitivity to seeing and drive errors, compared to a filar micrometer. The way the images are formed also permits comparative photometry to be carried out, thus ϑ, ρ & Δm may be measured.

OPTICAL PRINCIPLES of the LYOT MICROMETER

The only optical component is a plate carved from a single uniaxial crystal of Icelandic rock spar or calcite ($CaCO_3$) so that the angle between the crystallographic axis and the surface of the plate does not exceed 1˚. Deviation from planarity has to be less than $\frac{1}{4}\lambda$ in any circle of 4mm radius (the plate thickness). The plate must be parallel within 3' arc. Because the crystalline material is hygroscopic, the surfaces are coated with a dielectric vacuum deposition of magnesium fluoride.

Because the calcite is a single uniaxial crystal it is birefringent. When inclined to the optical axis the randomly polarised incident ray becomes linearly polarised in two mutually perpendicular planes. Each plane of polarisation has a slightly different refractive index, referred to as, the ordinary and the extraordinary. So the ray is effectively split into two, almost parallel rays. The departure from collimation is dependent upon the incident ray height. The focal ratio must be less than *f*/6, and preferably *f*/15. For accurate photometry it must exceed *f*/20.

Referring to fig.1, which depicts the geometry of the ray path through the spar plate (spath blade), the image heights depend on the incident ray height at the first surface. Using parallel plate theory, the angular distance function is given by the separation of the ray heights of the ordinary and extraordinary images:

ref. fig. 1

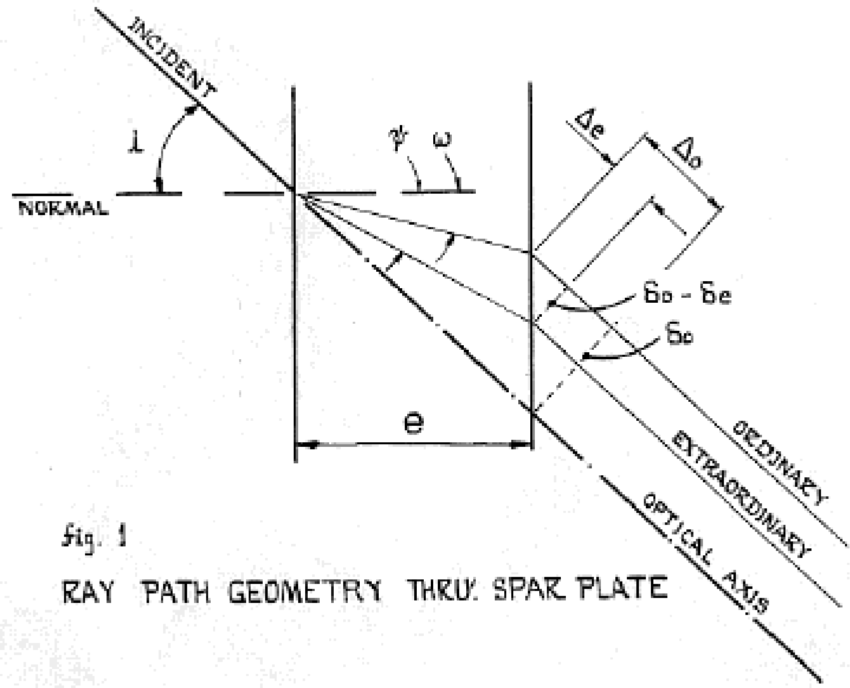


fig. 1

RAY PATH GEOMETRY THRU'. SPAR PLATE

$$\delta_o - \delta_e = (\Delta_o - e)\sin\omega - (\Delta_e - e)\sin\psi$$

where Δ_o & Δ_e are lateral displacements along the optical axis
δ_o & δ_e are the ordinary & extraordinary ray heights
ω & ψ are the ordinary & extraordinary angles of refraction

but

$$\Delta_o = e\left(1 - \frac{\cos i}{\cos\omega}\right)$$

&

$$\Delta_e = e\left(1 - \frac{\cos i}{\cos\psi}\right)$$

where i is the angle of incidence (the measured inclination of the spath blade) hence:

$$\delta_o - \delta_e = e\sin\omega\left(1 - \frac{\cos i}{\cos\omega}\right) - e\sin\psi\left(1 - \frac{\cos i}{\cos\psi}\right) - e(\sin\omega - \sin\psi)$$

$$= e\cos i(\tan\psi - \tan\omega)$$

and taking e out as a constant to be used when converting from unit to scalar displacement:

the angular distance function $\phi(i) = \cos i(\tan\psi - \tan\omega)$

but
$$\tan\psi = \frac{\sin\psi}{\sqrt{1 - \sin^2\psi}}$$

substituting
$$\sin\psi = \frac{\sin i}{n_e}$$

$$\tan\psi = \frac{\sin i}{n_e\sqrt{1 - \frac{\sin^2 i}{n_e^2}}} = \frac{\sin i}{\sqrt{n_e^2 - \sin^2 i}}$$

and making a similar substitution for $\tan\omega$

$$\phi(i) = \cos i.\sin i\left[\frac{1}{\sqrt{n_e^2 - \sin^2 i}} - \frac{1}{\sqrt{n_2^o - \sin^2 i}}\right]$$

but
$$\cos i.\sin i = \tfrac{1}{2}\sin 2i \quad \& \quad \cos 2i = 1 - 2\sin^2 i$$

thus
$$\phi(i) = \frac{\sin 2i}{\sqrt{2}}\left[\frac{1}{\sqrt{2n_e^2 - 1 + \cos 2i}} - \frac{1}{\sqrt{2n_o^2 - 1 + \cos 2i}}\right]$$

and converting to arcsecs
$$\phi(i) = \frac{648000}{\pi}\frac{\sin 2i}{\sqrt{2}}\left[\frac{1}{\sqrt{2n_e^2 - 1 + \cos 2i}} - \frac{1}{\sqrt{2n_o^2 - 1 + \cos 2i}}\right]$$

The scalar conversion is given by $d = \frac{e}{F}\phi(i)$ where e is the blade thickness and F the effective focal length. (ρ is a function of d, to be explained later - see section dealing with the physical description of the micrometer and its use).

I have plotted the ADF, ϕ(i) (ref. fig.2). It should be noted that n_e & n_o are not constant. The refractive indices vary with temperature and wavelength. The working values at 18˚C & a wavelength of 589.29nm (Na-D line). are $n_e = 1.48639$ & $n_o = 1.65836$;

The thermal variation coefficients are $\Delta n_e = 1.18.10^{-5}$ & $\Delta n_o = 2.1.10^{-6}$ per˚C;

Dispersion is $\frac{d}{d\lambda}n_o = -7.003.10^{-6}.A^{-1}$ & $\frac{d}{d\lambda}n_e = -3.138.10^{-6}.A^{-1}$

It will be noted that the slope $\phi(i)$is at a maximum when i=0˚ and zero at i=55˚.8. The slope may be determined by taking the derivative of $\phi(i)$, holding n_e & n_o constant, and differentiating with respect to i; hence from:

$$\phi(i) = \tfrac{1}{2}\sin 2i\left[\left(n_e^2 - \sin^2 i\right)^{-\frac{1}{2}} - \left(n_o^2 - \sin^2 i\right)^{-\frac{1}{2}}\right]$$

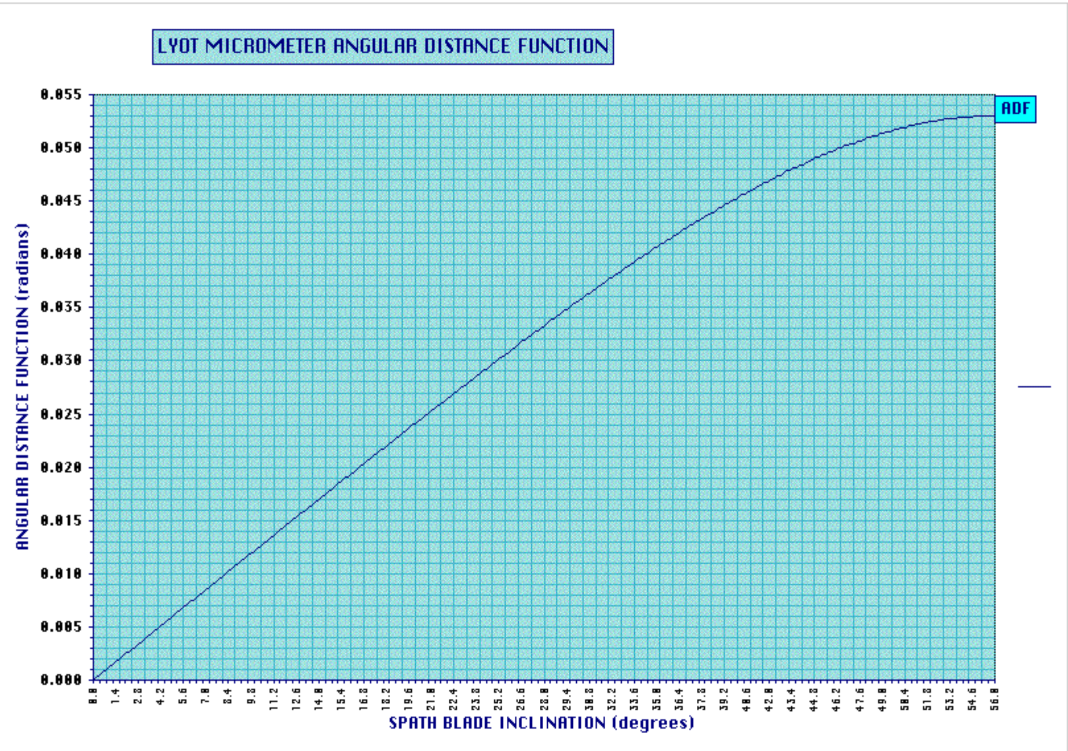
$$\frac{d}{di}\phi(i) = \cos 2i\left[\left(n_e^2 - \sin^2 i\right)^{-\frac{1}{2}} - \left(n_o^2 - \sin^2 i\right)^{-\frac{1}{2}}\right] + \tfrac{1}{8}(1-\cos 4i)\left[\left(n_e^2 - \sin^2 i\right)^{-\frac{3}{2}} - \left(n_o^2 - \sin^2 i\right)^{-\frac{3}{2}}\right]$$

$$when\ i = 0^{\circ} \quad \frac{d}{di}\phi(i) = 0.069765562$$

$$taking\ di = 0^{\circ}.1 \quad d\phi(i) = 1.218.10^{-4} rads = 25''.12 arc$$

The second order differential is given by:

$$\frac{d^2}{di^2}\phi(i) = \sin 2i\left\{\begin{array}{l}\left[\tfrac{1}{2}\cos 2i\left(n_e^2 - \sin^2 i\right)^{-\frac{3}{2}} - 2\left(n_e^2 - \sin^2 i\right)^{-\frac{1}{2}}\right] - \\ \left[\tfrac{1}{2}\cos 2i\left(n_o^2 - \sin^2 i\right)^{-\frac{3}{2}} - 2\left(n_o^2 - \sin^2 i\right)^{-\frac{1}{2}}\right] + \\ \left[\tfrac{3}{16}(1-\cos 4i)\left(n_e^2 - \sin^2 i\right)^{-\frac{5}{2}} + \cos 2i\left(n_o^2 - \sin^2 i\right)^{-\frac{3}{2}}\right] - \\ \left[\tfrac{3}{16}(1-\cos 4i)\left(n_o^2 - \sin^2 i\right)^{-\frac{5}{2}} + \cos 2i\left(n_o^2 - \sin^2 i\right)^{-\frac{3}{2}}\right]\end{array}\right\}$$



$\phi(i)_{\max}$ occurs when $\frac{d}{di}\phi(i) = 0$

taking a working temperature range of -10˚C to 30˚C, we obtain the maximum angular distance functions and the corresponding values of i:

$$t=-10^\circ C \quad i_{\max}=55^\circ.802$$
$$\phi(i)_{\max}=10952''.76\,arc$$
$$t=18^\circ C \quad i_{\max}=55^\circ.798$$
$$\phi(i)_{\max}=10930''.90\,arc$$
$$t=30^\circ C \quad i_{\max}=55^\circ.796$$
$$\phi(i)_{\max}=10921''.54\,arc$$

In practice the image becomes progressively degraded by chromatic aberration and astigmatism when $i \geq 40^\circ$.

@ $i=40^\circ, \quad \phi(i)=9344''.63arc \quad \& \quad \frac{d}{di}\phi(i)=17''.58arc\,(di=0^\circ.1)$

The value of φ(i) is temperature dependent. To investigate the thermal variation coefficient it is necessary to redefine f(i) in terms of the thermal coefficients of the refractive indices and differentiate with respect to the temperature difference from a selected datum.

rewriting:

$$\phi(i)=\tfrac{1}{2}\sin 2i\left[\left(\left(n_e+(t-t_0)\Delta n_e\right)^2-\sin^2 i\right)^{-\frac{1}{2}}-\left(\left(n_o+(t-t_0)\Delta n_o\right)^2-\sin^2 i\right)^{-\frac{1}{2}}\right]$$

$$\frac{d}{d(t-t_0)}=\tfrac{1}{2}\sin 2i\begin{bmatrix}\left(\Delta n_o^2(t-t_0)+n_o.\Delta n_o\right)\left(\left(n_o+(t-t_0)\Delta n_o\right)^2-\sin^2 i\right)^{-\frac{3}{2}}- \\ \left(\Delta n_e^2(t-t_0)+n_e.\Delta n_e\right)\left(\left(n_e+(t-t_0)\Delta n_e\right)^2-\sin^2 i\right)^{-\frac{3}{2}}\end{bmatrix}$$

$$where \quad t_0=18^\circ C \quad \Delta n_e=1.18.10^{-5} \quad \& \quad \Delta n_o=2.1.10^{-6}$$

Although the thermal variation coefficient varies with i & t in a non linear manner, the second order differentials in both i & t, fortunately, are almost linear.

The maximum thermal variation in φ(i) occurs at i_{max}
At t_0=18˚C and when

$$i=55^\circ.8 \quad \frac{d}{dt}\phi(i)=-0''.78arc$$

and when $\quad i=40^\circ \quad \frac{d}{dt}\phi(i)=-0''.64arc$

Although the angular distance function varies with wavelength, increasing towards the blue, the eye is so strikingly selective in its response that in practice the wavelength range is too restricted to make a sensible difference. The dispersion properties of Calcite and the effect it has upon the performance characteristics of the Meca- Precis Lyot micrometer is dealt with later.

THE MECA-PRECIS LYOT MICROMETER

The micrometer has three sections; the PA circle, which is a spur wheel mounted on the drawtube nosepiece; the spath blade, housed in a block shaped body; the eyepiece holder.

The PA circle is graduated in 20˚ intervals in large white numerals. The circle is driven by a pinion which has a graduated dial that reads, by vernier, to 0˚.1.

The spath blade housing can rotate around the axis on the PA wheel. The blade may also rotate on an axis perpendicular to the optical axis. The inclination of the blade to the optical axis is controlled by an anti-backlash spur gear and may be read from a separate dial and vernier to 0˚.1 over a 60˚ range. Both rotation and inclination movements enable ϑ & ρ to be measured.

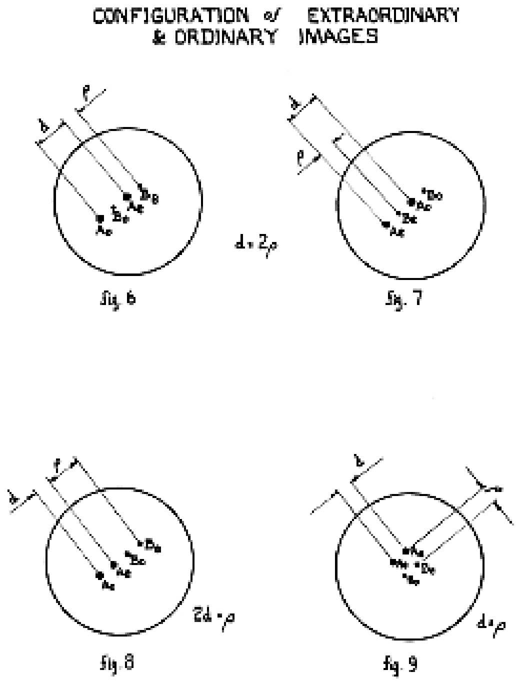
The entire micrometer is manufactured out of 2014-T4 aluminium alloy and finished in anodized satin black. All the graduations are in clear white lettering and very easy to read.

The eyepiece holder accepts either 31.75mm (American - $1\frac{1}{4}$ INCH) or 27mm (French - 1 Paris INCH) push fit series eyepieces. Any eyepiece may be used - a low power to acquire the field and then a medium to high power for the measure. An illuminated reticule is needed ***only*** to zero the PA, it is ***not*** needed for ϑ or ρ measures. Because there are no wires, their are no wire magnification effects, i.e. a focusing differential & parallax, and apparent & real subtended angular width. Because I prefer to sit at the eyepiece, I have installed a Vixen projection reticule guide eyepiece. Regardless of the PA orientation of the micrometer body I can always look down into the eyepiece, rather like looking into a microscope. This means my eye orientation is fixed and astigmatic effects are eliminated. The PA is zeroed with the projection reticule switched on, and then it is switched off for the remainder of the session. There are no wires to clutter the field of view. A Barlow may be fitted in the drawtube nosepiece. I have obtained a coated triplet negative apochromat. It fits against a register so the amplification is fixed.

USING the LYOT MICROMETER

To zero the PA a single star is separated by inclining the spath blade to 40° and both images aligned to an equatorial reticule web. To eliminate centring error the same method as used for a filar micrometer is employed. The PA readings are taken using the double difference method.

Measurement of the separation is a similar process to that employed using a diffraction micrometer. The images are arranged in a regular alignment. Six cases may arise (ref. figs. 6,7,8 & 9). The alignments need not arise by chance, it is possible to adopt the square alignment for the ρ measure and one of the four possible linear alignments (dependent upon the separation) for the ϑ measure. As when using a filar micrometer, although it is possible to make ρ & ϑ measures at the same time, it may be desirable to split them into distinct operations.



REDUCING the SEPARATION MEASURE

The initial difficulty lies in determining the $\frac{e}{F}$ ratio. The screw constant of a filar micrometer is given by:

$$\vartheta = \frac{648000}{\pi} \cdot \frac{p}{F} \qquad \text{where p is the screw pitch}$$

When restricting measures to less than 30" arc it is possible to obtain a good estimate of J by adopting the manufacturer's quoted pitch value. All one then needs to ascertain is the effective focal length, F, and there are several tried and tested methods for doing so with the necessary precision. My preference is to measure declination differences of HIC stars in M45. This cannot be done with the Lyot micrometer because it cannot measure wide separations.

CALIBRATING the MICROMETER on KNOWN PAIRS

The manufacturer quotes e as being 4.0mm ±0.1mm. Adopting the quoted mean, and taking a previously calibrated value of F (3542.65mm):

$$\frac{e}{F} = \frac{4}{3542.65} = 1.1291.10^{-3}$$

The $\frac{e}{F}$ ratio may be refined by calibrating the micrometer upon pairs of known separation:

e.g. ζBOO 31 MAY94 t=10˚C ρ = 6”.88 (1994.5)

2i = 22.5 + 60 - 37.3 = 45.2
2i = 23.7 + 60 - 36.1 = 47.6
2i = 23.7 + 60 - 39.3 = 44.4
2i = 25.1 + 60 - 35.7 = 49.4
2i = 25.3 + 60 - 34.4 = 50.9
2i = 26.0 + 60 - 35.2 = 50.8
$\sigma_n = 2.555$ i = 24.025

$\phi(i) = 5938.44$

$\rho = 6''.71 \pm 0''.23 \quad (\textit{probable error @ 50\% confidence level})$

πBOO 06JUN94 t=10˚C r = 5”.27 (Argyle 1993.2)

2i = 15.3 + 60 - 38.5 = 36.8
2i = 15.9 + 60 - 37.9 = 38.0
2i = 14.8 + 60 - 37.6 = 37.2
2i = 15.8 + 60 - 37.0 = 38.8
$\sigma_n = 0.768$ i = 18.85

$\phi(i) = 4695.7$

$\rho = 5''.30 \pm 0''.08$

πBOO 06JUN94 mean of 6 measures ρ = 5”.40+0”.03 -0”.06
πBOO 11JUN94 mean of 6 measures ρ = 5”.35±0”.07
πBOO 13JUN94 mean of 6 measures ρ = 5”.35±0”.05
mean of 22 measures over 4 nights ρ = 5”.35±0”.075

σCRB 14JUN94 t=12˚C ρ = 7”.00 (1994.5)
mean of 6 measures ρ = 6”.96±0”.03

Both sets of measures lie, within the 50% confidence level, sufficiently close to the calibration to warrant its adoption. However it is possible to use the calibrated separations to rework F so that they more closely proximate their predicted values.

PROBABLE ERROR of SEPARATION MEASURE

The 50% confidence level (assuming a normal distribution) is known as the probable error, and is defined by:

where d = ρ $\quad \rho = \frac{e}{F}\phi\left(\bar{i} \pm 0.6745\sigma_i\right)$

where $\bar{i}$ is the mean of inclination measures

σ_i is the sample standard deviation of the mean

putting error in $\bar{i}$, $\Delta i = \pm 0.6745\sigma_i$

then error is ρ:

$$\Delta\rho = \pm\frac{e}{F}\left\{\phi\left(\bar{i} + \Delta i\right) - \phi\left(\bar{i}\right)\right\}$$

in the limit

$$\Delta\phi\left(\bar{i}\right) = \left\{\phi\left(\bar{i} + \Delta i\right) - \phi\left(\bar{i}\right)\right\} = \Delta i.\frac{d}{di}\phi\left(\bar{i}\right)$$

hence

$$\Delta\rho = \frac{e}{F}\Delta i.\frac{d}{di}\phi\left(\bar{i}\right)$$

and the second order error (due to rate of change

$$\frac{d}{di}\Delta\rho = \pm\frac{e}{F}.\Delta i.\frac{d^2}{di^2}\phi\left(\bar{i}\right)$$

COMPARISON of CALIBRATION ACCURACY with a FILAR MICROMETER

The Lyot micrometer was calibrated using an absolute method of my own devising. There was a good reason for devising the method. Attempting to make an initial calibration by measuring a known bright pair at $\approx 5''arc$ sep. would incur a significant calibration error. Errors in calibration add to errors to measurement. Providing the calibration error is at least an order of magnitude smaller than the measurement error, it is not significant; it cannot effect the error calculated from the probable error of the sample standard deviation of the mean.

A filar micrometer, provided it has adequate travel, has a decided advantage in that the separation calibration may be made over an arc of $\frac{1}{2}^\circ$ or so, and the screw constant averaged over the number of revolutions needed to encompass the separation. It is therefore not necessary to employ an absolute method when calibrating a filar micrometer.

taking $d = \frac{e}{F}\phi(i)$

the measurement error $\Delta d = -\frac{\Delta F.e.\phi(i)}{F^2}$

and taking $R = \frac{648.10^{-3}}{\pi}.\frac{p}{F}$

where p is the screw pitch

the measurement error $\Delta R = -\Delta F.\frac{648.10^{3}}{\pi}.\frac{p}{F^2}$

comparing errors $\frac{\Delta d}{\Delta R} = \frac{p}{e.\phi(i)}$ where $\phi(i)$ is expressed in radians

$@\phi(i)_{\max}$ $\rho_{\max} = \frac{e}{F}\phi(i)_{\max}$ where $\rho = d$

$\frac{\rho}{R} = \frac{e.\phi(i)}{p}$ where $\phi(i)$ is expressed in radians

hence $\rho\Delta d = R\Delta R$

thus when ΔR is factored to equate to Δd, the ΔF errors are equal.

OPERATING RANGE & ERRORS of SEPARATION MEASURES

In describing the optical and operational characteristics of the Lyot micrometer I have dealt with calibration and separation errors in isolation. I have also only touched upon those factors that restrict the upper operating limit, namely the optical limit, $i \cong 55^{\circ}.8$ and error rate limit. A full description of the micrometer's operational characteristics would not be complete without an analysis of the way the error rate and calibration error effect the estimate of the separation error, and ultimately impose an upper operating limit somewhat less than the optical limit.

Referring to the upper limit of the operating range, it was stated that beyond $i \geq 40^{\circ}$, the image becomes progressively degraded by astigmatism and chromatic aberration. This is only partly the reason why separations beyond $i \approx 40^{\circ}$ become increasingly inaccurate. The Seidel errors do not become really troublesome until $\iota \geq 50^{\circ}$ when the micrometer is used at focal ratios slower than $f/20$. Within the optically defined range $\left(0^{\circ} \leq i \leq 55^{\circ}.8\right)$, the error rate gradually increases. The measurement error is a linear differential function of the probable error of the measures; the error rate over the range of measures, plus the calibration error.

hence: Sum of measurement errors:

$$\sum_{0}^{\Delta i} \Delta\rho = \frac{e}{F}.\Delta i.\frac{d}{di}\phi\left(\bar{i}\right) + \frac{e}{F}.\Delta i.\frac{d^2}{di^2}\phi\left(\bar{i}\right) + \kappa.\phi\left(\bar{i}\right)$$

where the calibration error

$$\kappa = 2\left\{\frac{e}{F} - \frac{e}{F + \Delta F}\right\}$$

in practice Δi is also a function of i; the lower the inclination of the plate, the more confined the spread in measures.

I have modelled Δi over the operating range and find a very good approximation is given by the power function:

$$\Delta i = 0.6745.e^{s}.\left(1 + 0.1\bar{i}\right)^{s}$$

where s is a constant whose value is dependent on the seeing:

SEEING SPURIOUS DISC RADα	S
$I \leq 0.25\alpha$	0.1
$II \geq 0.25\alpha$	0.25
$III \leq 0.50\alpha$	0.50
$IV \leq 1.00\alpha$	1.00
$V \geq 1.50\alpha$	1.50

α spurious disc radius in units of $1.22\lambda f/\#$

EYEPIECE DISTORTION & SYSTEMATIC ERROR in POSITION ANGLE

Eyepiece distortion if present will produce a significant systematic error. Distortion is proportional to θ^3 (field angle) and does not depend on the ray height. For a set of point objects equally spaced perpendicular to the optical axis, the set of images will not be equally spaced if distortion is present. If therefore ϑ is determined using the double difference method in the presence of distortion, the difference between the mean 'before' and 'after' turning measures will not be 180˚. In fact the error caused by aligning the images 'before turning' will be doubled.

It is imperative therefore that a distortion free eyepiece be used. There is one zero distortion eyepiece design, the modified Ramsden manufactured by Augen Optics, that is to be especially commended.

Distortion due to a plano-convex lens is given by:

$$DI = A_3\theta^3$$

$$A_3 = -\frac{1}{8}\left[\Gamma\left(\frac{1}{S}-\frac{1}{R}\right)^2 + \frac{K}{R^2}(n'-n) + b\right]$$

$$\Gamma = n^2\left(\frac{1}{n'\,s'}-\frac{1}{ns}\right)$$

for a spherical surface K = 0 & b = 0

for the modified Ramsden: n = 1 & $\frac{n'-n}{R} = -\frac{n}{f_e}$

The distortion introduced by the field lens is compensated by the eye lens. Furthermore the curvature of field is almost zero. The modified Ramsden is the ideal eyepiece for use with the Lyot micrometer.

DISPERSION of CALCITE & the VARIATION of the ANGULAR DISTANCE FUNCTION

In a negative uniaxial crystal like Calcite, the extraordinary index of refraction is less than the ordinary index. Both indices increase as the wavelength decreases, but the ordinary index increases more than the extraordinary index, hence the index difference (μ) increases. The dispersion characteristics of Calcite in the visible region (400nm to 700nm) may be modelled by the Cauchy three term equation:

$$n_o = A_o + B_o.\lambda^{-2} + C_o.\lambda^{-4}$$
$$n_e = A_e + B_e.\lambda^{-2} + C_e.\lambda^{-4}$$

the terms are derived by solving simultaneous equations set up from known values of n_o & n_e at three differents λ's:
@18˚C

WAVELENGTH	n_o	n_e	μ
$\hat{A}$ 4046.56	1.68134	1.49694	-0.18440
5892.90	1.65836	1.48639	-0.17197
7065.20	1.65207	1.48359	-0.16848

from which

$$A_o = 1.637\,610\,656$$
$$B_o = 727\,557.849\,5$$
$$C_o = -1.392\,703\,258.10^{11}$$
$$A_e = 1.477\,292\,983$$
$$B_e = 310\,723.589\,6$$
$$C_e = 1.799\,474\,718.10^{11}$$

Between 400nm & 700nm the Cauchy three term equations represent n_o & n_e to 5 dec. plcs.

Because $\phi(i)$ varies with λ, being greater in blue than red light, as the spar plate is tilted, a star is drawn out into a very short spectrum. The angular length of the spectrum between the C & F lines is given by:

$$f/\# = \left(\phi(i)_{\lambda_C} - \phi(i)_{\lambda_F}\right)$$

At $i = 40^\circ$ $\phi(i)$ varies between the C & F lines by 1.186.10^{-3} rads. It can be readily demonstrated that the dispersion is less than the diameter of the Airy disc:
the limiting case occurs where $f/\# = \frac{e}{138.4}\left(\phi(i)_{\lambda_C} - \phi(i)_{\lambda_F}\right) = f/7.1$ which is less than half the lower operating limit of $f/15$.