Recalculation of the Rev. James Challis' true pole alignment method for equatorials

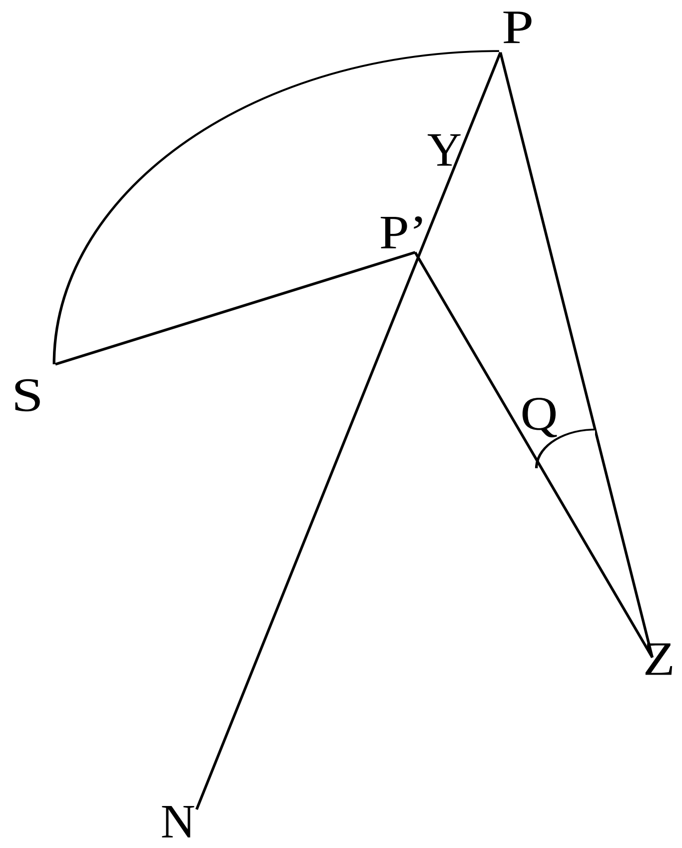
PP' = γ = POLAR DISTANCE OF P'

ZPP' = ϑ = HOUR ANGLE OF P'

LET δ' = INSTRUMENTAL DECLINATION

LET δ_o = TRUE DECLINATION

LET τ = TRUE HOUR ANGLE



γ & ϑ MAY BE DETERMINED FROM:

$$\sin\delta_1' = \sin\delta_1 \cos\gamma + \cos\delta_1 \sin\gamma \cos(\tau_1 - \vartheta)$$
$$\sin\delta_2' = \sin\delta_2 \cos\gamma + \cos\delta_2 \sin\gamma \cos(\tau_2 - \vartheta)$$

QSP = ϑ = TRUE POLAR ANGLE

QSP' = ϑ' = INSTRUMENTAL POLAR ANGLE

PSP' = λ = CORRECTION

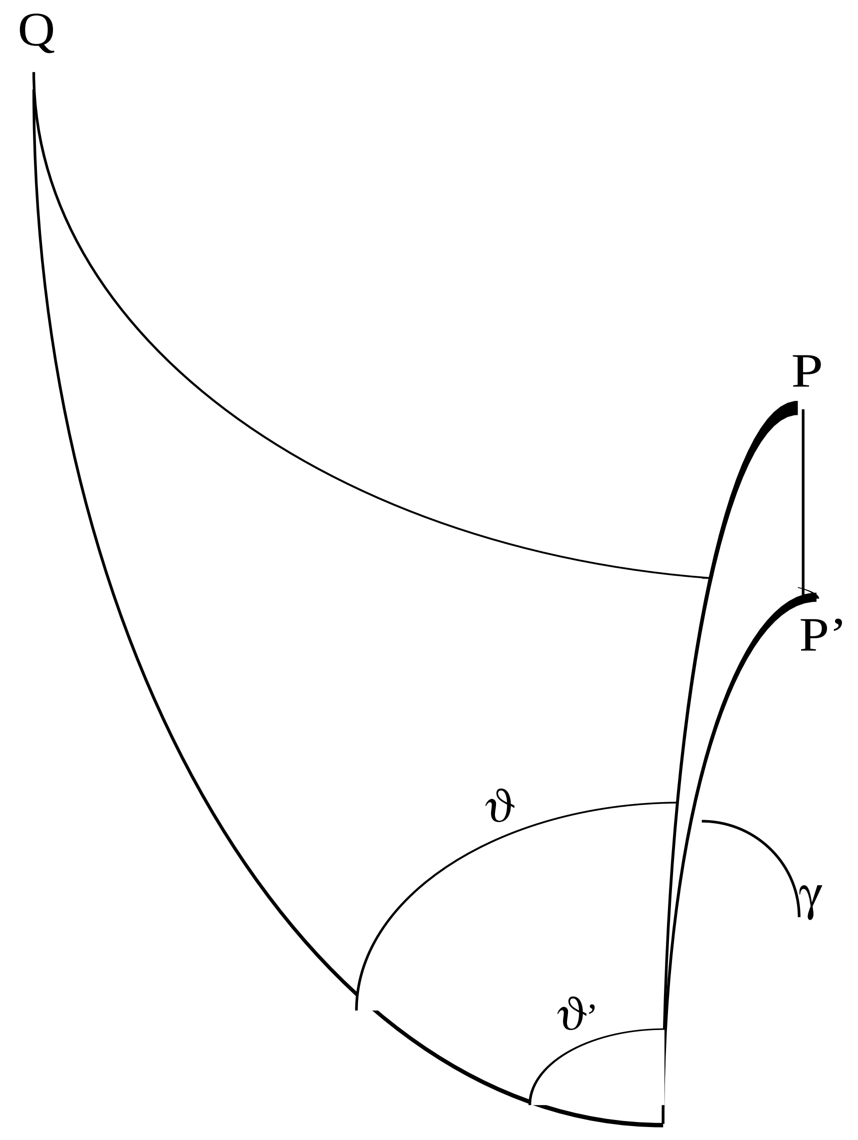
Q = POLE OF DECLINATION AXIS

i = DECLINATION OF Q

c = COLLIMATION ERROR (RADIANS)

$$\theta = \theta' - \lambda$$

$$\lambda = \gamma \sin(\tau - \vartheta) \sec\delta - i \sec\delta + c \tan\delta$$



NOTE: i = the departure from perpendicularity by the declination axis from the hour axis

CHALLIS' DECLINATION DRIFT METHOD:

AZIMUTH OF INSTRUMENTAL POLE FROM TRUE POLE: $\eta = \gamma \sin\vartheta$

ALTITUDE OF INSTRUMENTAL POLE FROM TRUE POLE: $\xi = -\gamma \cos\vartheta$

LET

$$\alpha = \delta_2' - \delta_1' \quad @ \quad \tau \ \& \ \tau_1 \quad *$$

$$\beta = \delta_3' - \delta_2' \quad @ \quad \tau_1 \ \& \ \tau_2 \quad *$$

* Dec drifts corrected for refraction

$$\therefore \alpha = \xi\left(\cos\tau_1 - \cos\tau\right) + \eta\left(\sin\tau_1 - \sin\tau\right)$$

$$\therefore \beta = \xi\left(\cos\tau_2 - \cos\tau\right) + \eta\left(\sin\tau_2 - \sin\tau\right)$$

OBTAINING ξ BY DIVIDING BY COEFFICIENT OF η

$$\frac{\alpha}{(\sin\tau_1 - \sin\tau)} = \xi\frac{(\cos\tau_1 - \cos\tau)}{(\sin\tau_1 - \sin\tau)} + \eta$$

$$\frac{\beta}{(\sin\tau_2 - \sin\tau)} = \xi\frac{(\cos\tau_2 - \cos\tau)}{(\sin\tau_2 - \sin\tau)} + \eta$$

$$\frac{\beta}{(\sin\tau_2 - \sin\tau)} - \frac{\alpha}{(\sin\tau_1 - \sin\tau)} = \xi\frac{(\cos\tau_2 - \cos\tau)}{(\sin\tau_2 - \sin\tau)} - \xi\frac{(\cos\tau_1 - \cos\tau)}{(\sin\tau_1 - \sin\tau)}$$

$$\frac{\beta(\sin\tau_1 - \sin\tau) - \alpha(\sin\tau_2 - \sin\tau)}{(\sin\tau_2 - \sin\tau)(\sin\tau_1 - \sin\tau)} = \frac{\xi(\cos\tau_2 - \cos\tau)(\sin\tau_1 - \sin\tau) - \xi(\cos\tau_1 - \cos\tau)(\sin\tau_2 - \sin\tau)}{(\sin\tau_2 - \sin\tau)(\sin\tau_1 - \sin\tau)}$$

$$\xi = \frac{\beta(\sin\tau_1 - \sin\tau) - \alpha(\sin\tau_2 - \sin\tau)}{(\cos\tau_2 - \cos\tau)(\sin\tau_1 - \sin\tau) - (\cos\tau_1 - \cos\tau)(\sin\tau_2 - \sin\tau)}$$

EXPANDING DENOMINATOR & ARRANGING IN COMPLEMENTARY TERMS

$$(\cos\tau_2 - \cos\tau)(\sin\tau_1 - \sin\tau) - (\cos\tau_1 - \cos\tau)(\sin\tau_2 - \sin\tau) = \cos\tau_2\sin\tau_1 - \cos\tau_1\sin\tau_2 - \cos\tau_2\sin\tau + \cos\tau\sin\tau_2 - \cos\tau\sin\tau_1 + \cos\tau_1\sin\tau$$

$$= \tfrac{1}{2}\sin(\tau_1 - \tau_2) - \tfrac{1}{2}\sin(\tau_2 - \tau_1) + \tfrac{1}{2}\sin(\tau_2 - \tau) - \tfrac{1}{2}\sin(\tau - \tau_2) + \tfrac{1}{2}\sin(\tau - \tau_1) - \tfrac{1}{2}\sin(\tau_1 - \tau)$$

$$= \sin(\tau_2 - \tau) - \sin(\tau_2 - \tau_1) - \sin(\tau_1 - \tau)$$

SUBSTITUTING SIMPLIFIED DENOMINATOR & SIMPLIFYING NUMERATOR:

$$\xi = \frac{\beta \cos\tfrac{1}{2}(\tau_1 + \tau)\sin\tfrac{1}{2}(\tau_1 - \tau) - \alpha\cos\tfrac{1}{2}(\tau_2 + \tau)\sin\tfrac{1}{2}(\tau_2 - \tau)}{\tfrac{1}{2}\sin(\tau_2 - \tau) - \tfrac{1}{2}\sin(\tau_2 - \tau_1) - \tfrac{1}{2}\sin(\tau_1 - \tau)}$$

SIMILARLY:

$$\eta = \frac{\beta \sin\tfrac{1}{2}(\tau_1 + \tau)\sin\tfrac{1}{2}(\tau_1 - \tau) - \alpha\sin\tfrac{1}{2}(\tau_2 + \tau)\sin\tfrac{1}{2}(\tau_2 - \tau)}{\tfrac{1}{2}\sin(\tau_2 - \tau) - \tfrac{1}{2}\sin(\tau_2 - \tau_1) - \tfrac{1}{2}\sin(\tau_1 - \tau)}$$

BUT:

$$\tfrac{1}{2}\sin(\tau_2 - \tau) - \tfrac{1}{2}\sin(\tau_2 - \tau_1) - \tfrac{1}{2}\sin(\tau_1 - \tau) = -2\sin\tfrac{1}{2}(\tau_2 - \tau)\sin\tfrac{1}{2}(\tau_2 - \tau_1)\sin\tfrac{1}{2}(\tau_1 - \tau)$$

SUBSTITUTING DENOMINATOR:

$$\therefore \xi = \frac{\beta \cos\tfrac{1}{2}(\tau_1 + \tau)\sin\tfrac{1}{2}(\tau_1 - \tau) - \alpha\cos\tfrac{1}{2}(\tau_2 + \tau)\sin\tfrac{1}{2}(\tau_2 - \tau)}{2\sin\tfrac{1}{2}(\tau_2 - \tau)\sin\tfrac{1}{2}(\tau_2 - \tau_1)\sin\tfrac{1}{2}(\tau_1 - \tau)}$$

&

$$\therefore \eta = \frac{\beta \sin\tfrac{1}{2}(\tau_1 + \tau)\sin\tfrac{1}{2}(\tau_1 - \tau) - \alpha\sin\tfrac{1}{2}(\tau_2 + \tau)\sin\tfrac{1}{2}(\tau_2 - \tau)}{2\sin\tfrac{1}{2}(\tau_2 - \tau)\sin\tfrac{1}{2}(\tau_2 - \tau_1)\sin\tfrac{1}{2}(\tau_1 - \tau)}$$

&

$$\therefore \vartheta = \tan^{-1}\frac{-\xi}{\eta} \quad \& \quad \gamma = \sqrt{\xi^2 + \eta^2}$$