

## Dome Slit Synchronization

From equation (5.1-2), unit vector  $\frac{\overrightarrow{QS}}{|\overrightarrow{QS}|}$  is,

$$\frac{\overrightarrow{QS}}{|\overrightarrow{QS}|} = \begin{pmatrix} L_e' \\ M_e' \\ N_e' \end{pmatrix} = \begin{pmatrix} \cos \delta \cos(-H) \\ \cos \delta \sin(-H) \\ \sin \delta \end{pmatrix}$$

Vector P→Q in  $X_e'-Y_e'-Z_e'$  coordinate is,

$$\overrightarrow{PQ} = \begin{pmatrix} X_e' \\ Y_e' \\ Z_e' \end{pmatrix} = r \begin{pmatrix} \sin(H - \pi) \\ \cos(H - \pi) \\ 0 \end{pmatrix}$$

From equation (5.1-4), vector O→S in  $X_{dome}-Y_{dome}-Z_{dome}$  coordinate is,

$$\begin{aligned} \overrightarrow{OS} &= \begin{pmatrix} X_{dome} \\ Y_{dome} \\ Z_{dome} \end{pmatrix} = \begin{bmatrix} \cos\left(\phi - \frac{\pi}{2}\right) & 0 & \sin\left(\phi - \frac{\pi}{2}\right) \\ 0 & 1 & 0 \\ -\sin\left(\phi - \frac{\pi}{2}\right) & 0 & \cos\left(\phi - \frac{\pi}{2}\right) \end{bmatrix} \left[ \begin{pmatrix} r \sin(H - \pi) \\ r \cos(H - \pi) \\ 0 \end{pmatrix} + k \begin{pmatrix} \cos \delta \cos(-H) \\ \cos \delta \sin(-H) \\ \sin \delta \end{pmatrix} \right] + \begin{pmatrix} X_{dome0} \\ 0 \\ Z_{dome0} \end{pmatrix} \\ &= \begin{pmatrix} X_{dome0} + r \cos\left(\phi - \frac{\pi}{2}\right) \sin(H - \pi) + k \left( \cos\left(\phi - \frac{\pi}{2}\right) \cos \delta \cos(-H) + \sin\left(\phi - \frac{\pi}{2}\right) \sin \delta \right) \\ r \cos(H - \pi) + k \cos \delta \sin(-H) \\ Z_{dome0} - r \sin\left(\phi - \frac{\pi}{2}\right) \sin(H - \pi) + k \left( -\sin\left(\phi - \frac{\pi}{2}\right) \cos \delta \cos(-H) + \cos\left(\phi - \frac{\pi}{2}\right) \sin \delta \right) \end{pmatrix} \\ &= \begin{pmatrix} A + Dk \\ B + Ek \\ C + Fk \end{pmatrix} \end{aligned}$$

Where,

$k$ : Constant

$$A = X_{dome0} + r \cos\left(\phi - \frac{\pi}{2}\right) \sin(H - \pi)$$

$$B = r \cos(H - \pi)$$

$$C = Z_{dome0} - r \sin\left(\phi - \frac{\pi}{2}\right) \sin(H - \pi)$$

$$D = \cos\left(\phi - \frac{\pi}{2}\right) \cos \delta \cos(-H) + \sin\left(\phi - \frac{\pi}{2}\right) \sin \delta$$

$$E = \cos \delta \sin(-H)$$

$$F = -\sin\left(\phi - \frac{\pi}{2}\right) \cos \delta \cos(-H) + \cos\left(\phi - \frac{\pi}{2}\right) \sin \delta$$

Since point S is on the dome surface, length of vector  $O \rightarrow S$  is R.

$$(A + Dk)^2 + (B + Ek)^2 + (C + Fk)^2 = R^2$$

Then,

$$(D^2 + E^2 + F^2)k^2 + 2(AD + BE + CF)k - (R^2 - A^2 - B^2 - C^2) = 0$$

Solving this equation for k,

$$k = \frac{-(AD + BE + CF) + \sqrt{(AD + BE + CF)^2 + (D^2 + E^2 + F^2)(R^2 - A^2 - B^2 - C^2)}}{D^2 + E^2 + F^2}$$

$$\vec{OS} = \begin{pmatrix} X_{dome} \\ Y_{dome} \\ Z_{dome} \end{pmatrix} = \begin{pmatrix} A + Dk \\ B + Ek \\ C + Fk \end{pmatrix}$$

From equations (5.1-5) and (5.1-6), azimuth and elevation of dome slit are,

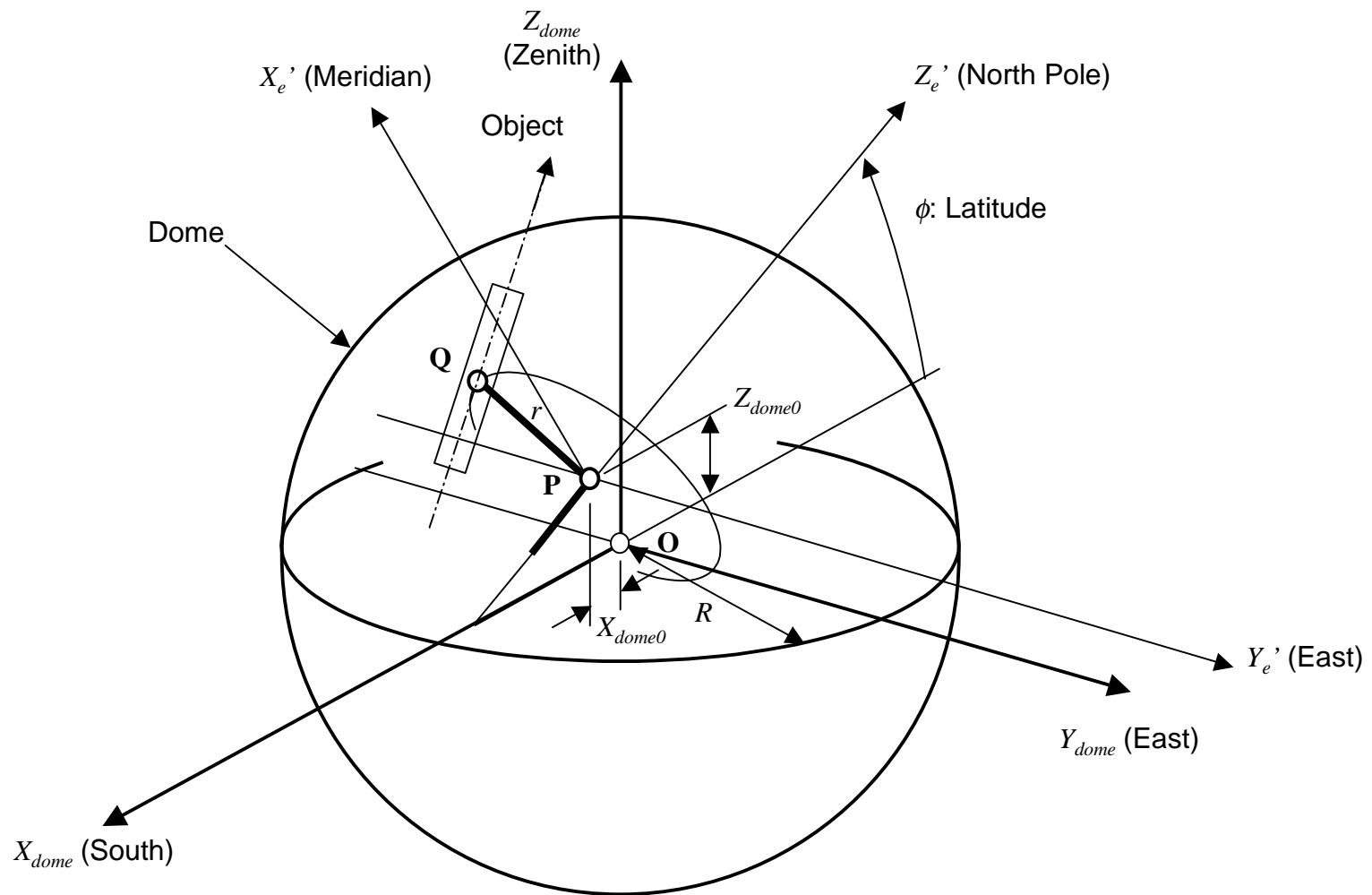
$$\tan(-A_{dome}) = \frac{Y_{dome}}{X_{dome}}$$

When  $X_{dome} \geq 0$ ,  $(-A_{dome})$  is in the 1st quadrant or the 4th quadrant.

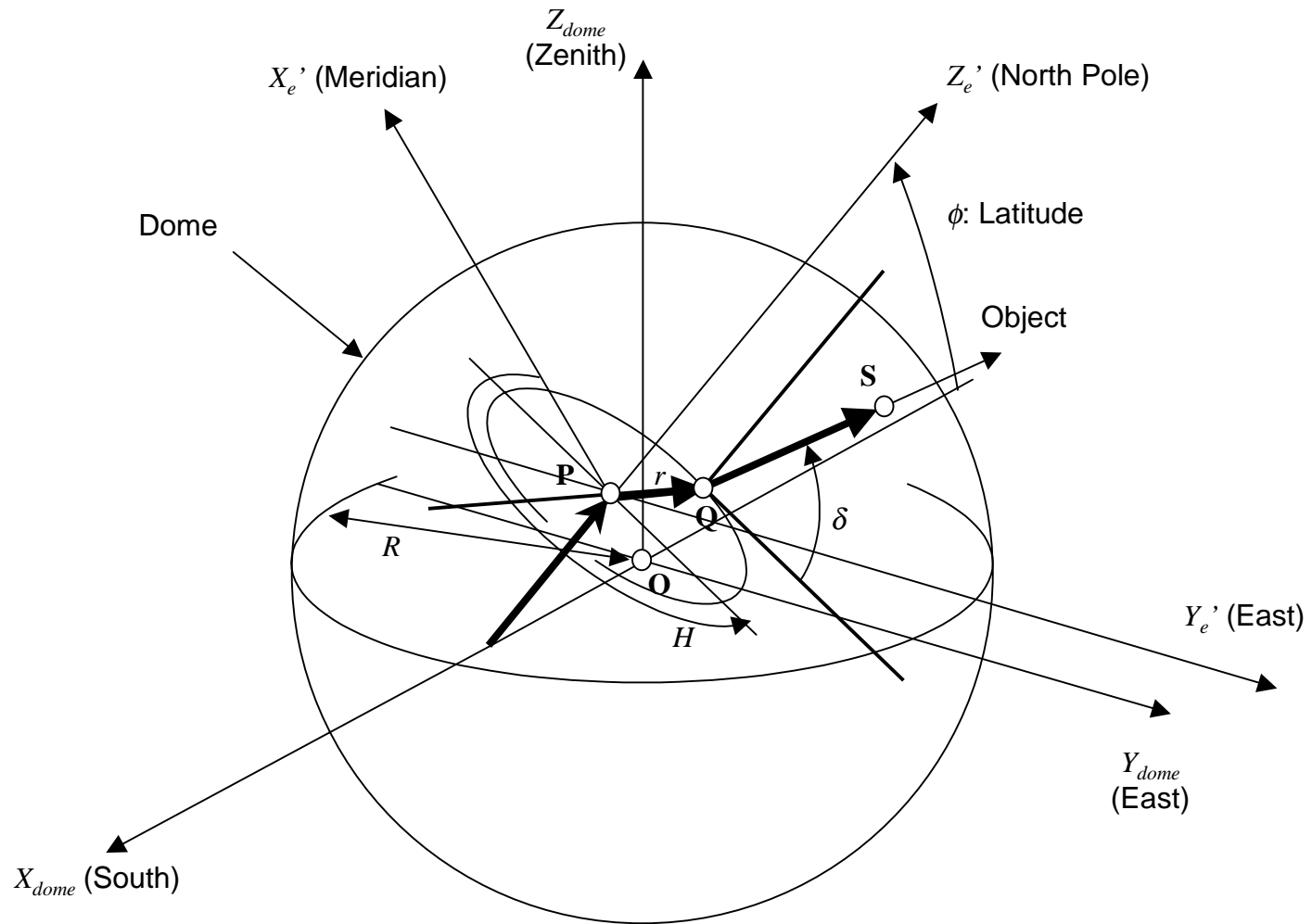
When  $X_{dome} < 0$ ,  $(-A_{dome})$  is in the 2nd quadrant or the 3rd quadrant.

$$\sin h_{dome} = \frac{Z_{dome}}{R}$$

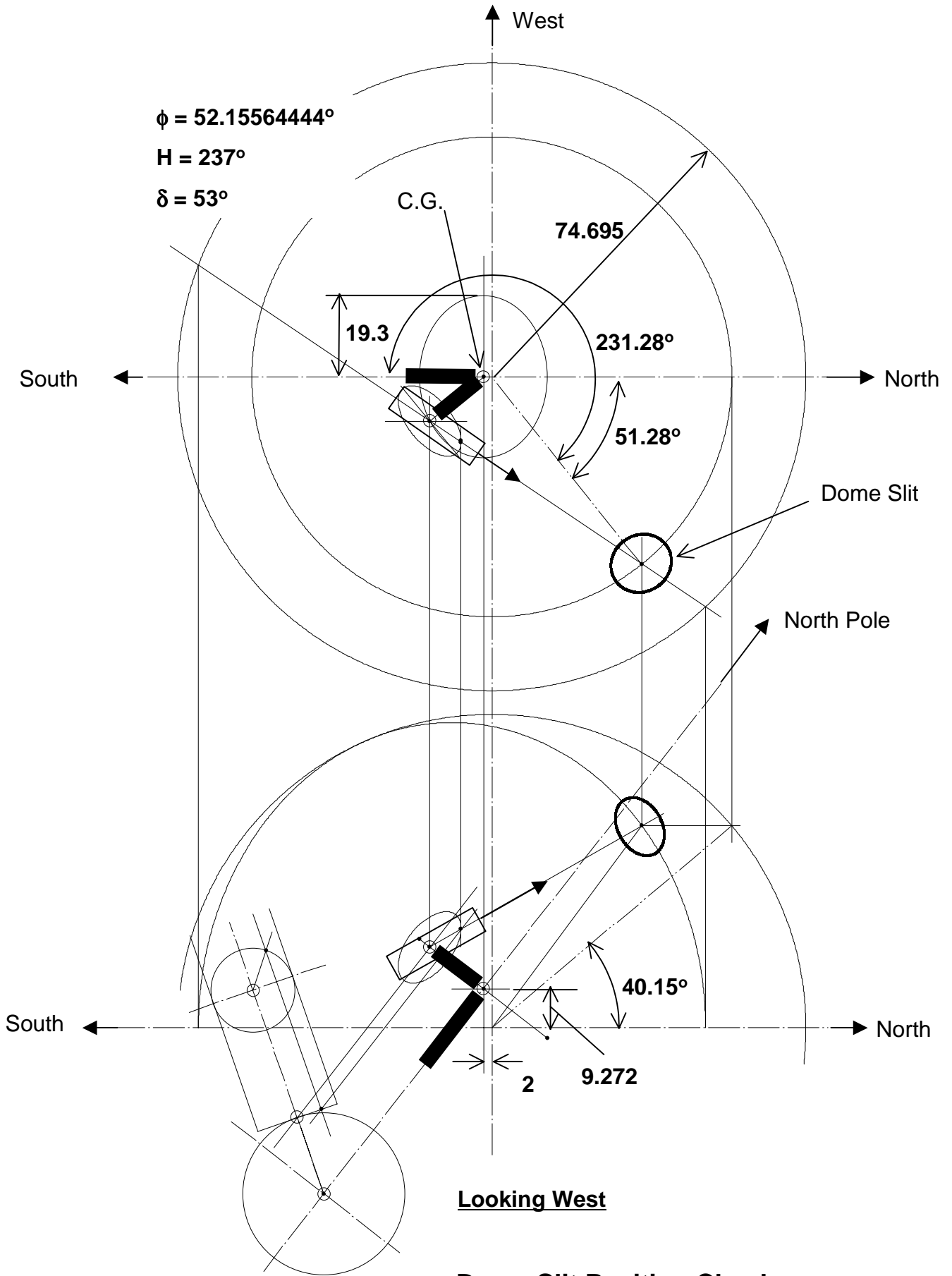
$$-\pi/2 (-90^\circ) \leq h_{dome} \leq +\pi/2 (+90^\circ)$$



**Figure 1. Definition of Coordinate Systems**



**Figure 2. Object in First Quadrant**



**Dome Slit Position Check**