Double star resolvability:

Relative performance of a 6-inch f/4 Newtonian and 2-inch f/12; 2.5-inch f/12; 3-inch f/16 & 4-inch f/16 OG

$S/\rho = 1.033 \lg_{10}^{-1} \frac{1}{n}(\Delta m - 0.1)$ where: S/ρ is the multiplication factor of Dawes' Limit $\frac{4''.56}{D_{ins}}$

n is the performance index

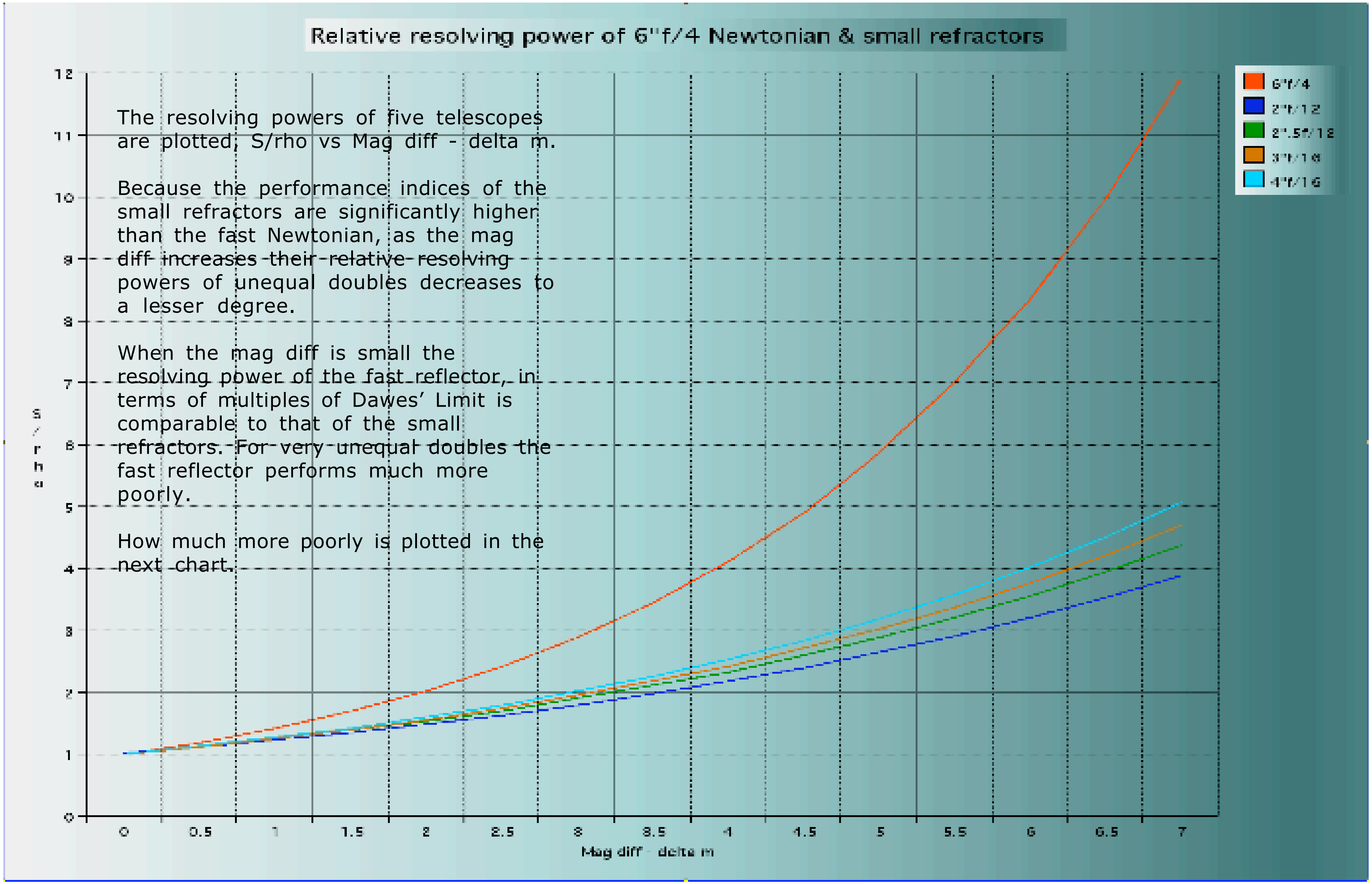
Δm is the mag diff

Δm	S/ρ 1 6″ f/4	S/ρ 2 2″ f/12	S/ρ 3 2″.5 f/12	S/ρ 4 3″ f/16	S/ρ 5 4″ f/16	RATIO 6″ / 2″	RATIO 6″ /2″.5	RATIO 6″ / 3″	RATIO 6″ / 4″	Δm limit
0	0.997	1.013	1.012	1.009	1.009	0.984	0.985	0.998	0.998	
1	1.421	1.228	1.247	1.271	1.271	1.157	1.140	1.118	1.118	
2	2.025	1.487	1.538	1.600	1.600	1.362	1.317	1.266	1.266	
3	2.886	1.802	1.896	2.014	2.014	1.602	1.522	1.433	1.433	4″ f/16 9
4	4.112	2.183	2.337	2.536	2.536	1.884	1.760	1.621	1.621	
5	5.861	2.645	2.881	3.192	3.192	2.216	2.034	1.836	1.836	3″ f/16 8
6	8.352	3.205	3.552	4.019	4.019	2.606	2.351	2.078	2.078	2″.5f/12 7
7	11.903	3.882	4.379	5.059	5.059	3.066	2.718	2.353	2.353	2″ f/12 6

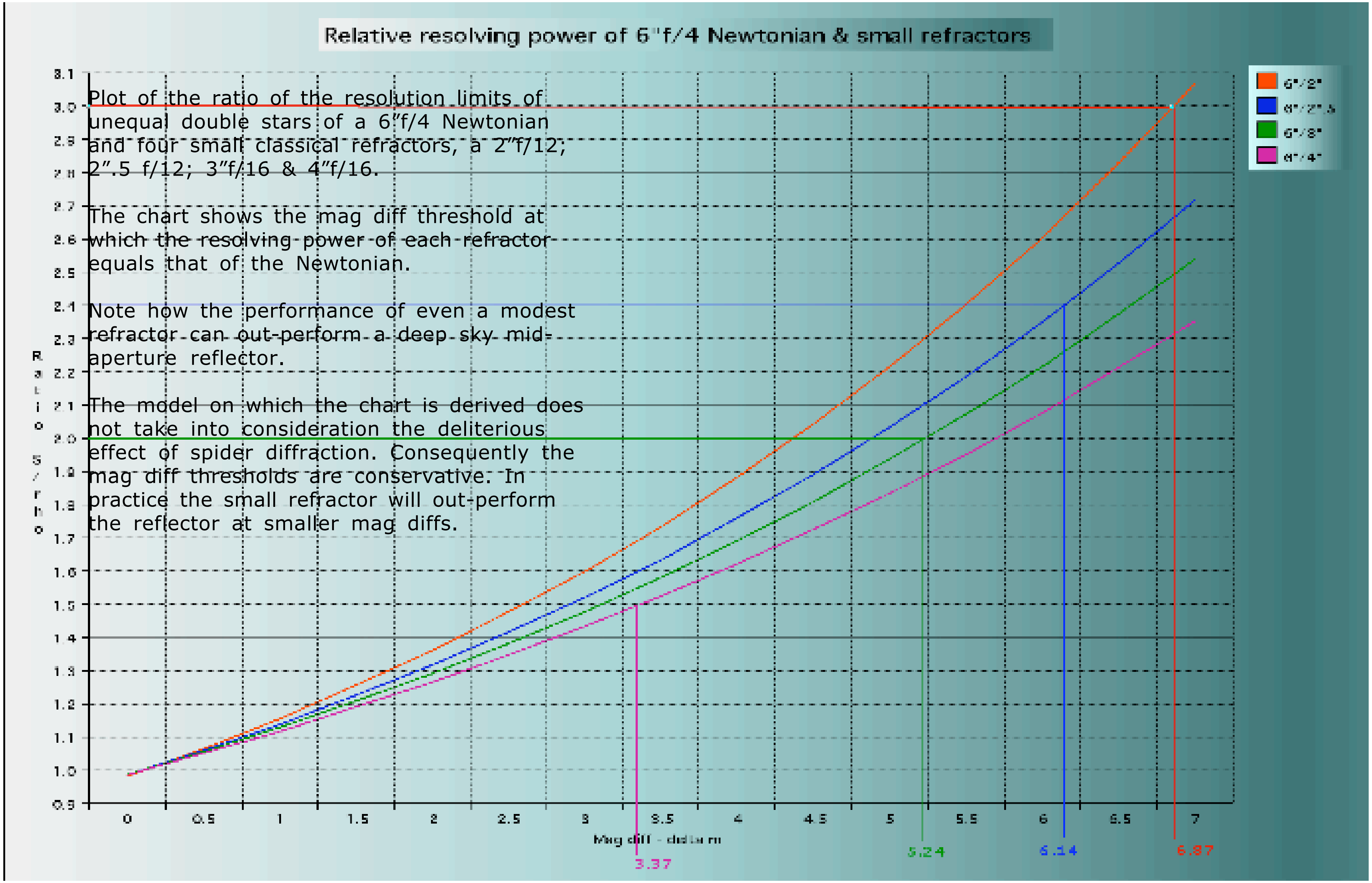
Ref:
1 - performance index (AIII) n = 6.5
2 - performance index (AI) n = 12
3 - performance index (AI-II) n = 11
4 - performance index (AII) n = 10.5
5 - performance index (AII-III) n = 10
6 - 2″f/12 equals 6″ f/4 @ $\Delta m = 6.87$
7 - 2″.5 f/12 equals 6″ f/4 @ $\Delta m = 6.14$
8 - 3″ f/16 equals 6″ f/4 @ $\Delta m = 5.24$
9 - 4″ f/16 equals 6″ f/4 @ $\Delta m = 3.37$

Because the small refractor has a higher performance index than the mid-aperture Newtonian, the S/ρ values for any particular mag diff Δm are smaller as Δm increases. When the ratio of the S/ρ values equals the ratio of the apertures, the corresponding Δm is the threshold beyond which the refractor will consistently out-perform the mid-aperture Newtonian.

NB This model ignores the deliterious effects of spider diffraction, and in practice the small refractor will out-perform the mid-aperture Newtonian at a lower mag diff.



Relative resolving power of 6"f/4 Newtonian & small refractors
The resolving powers of five telescopes are plotted, S/rho vs Mag diff - delta m.
Because the performance indices of the small refractors are significantly higher than the fast Newtonian, as the mag diff increases their relative resolving powers of unequal doubles decreases to a lesser degree.
When the mag diff is small the resolving power of the fast reflector, in terms of multiples of Dawes' Limit is comparable to that of the small refractors. For very unequal doubles the fast reflector performs much more poorly.
How much more poorly is plotted in the next chart.
6"f/4
2"f/12
2".5f/12
3"f/16
4"f/16
S/rho
Mag diff - delta m
0
1
2
3
4
5
6
7
8
9
10
11
12
0.5
1.5
2.5
3.5
4.5
5.5
6.5



Plot of the ratio of the resolution limits of unequal double stars of a 6"f/4 Newtonian and four small classical refractors, a 2"f/12; 2".5 f/12; 3"f/16 & 4"f/16.

The chart shows the mag diff threshold at which the resolving power of each refractor equals that of the Newtonian.

Note how the performance of even a modest refractor can out-perform a deep sky mid-aperture reflector.

The model on which the chart is derived does not take into consideration the deliterious effect of spider diffraction. Consequently the mag diff thresholds are conservative. In practice the small refractor will out-perform the reflector at smaller mag diffs.

GENERALISATION - RELATIVE RESOLVING POWERS of UNEQUAL DOUBLES

from: $$S/\rho_1 = 1.033\,\mathrm{lg}_{10}^{-1}\frac{1}{n_1}\left(\Delta m_t - 0.1\right)$$

& $$S/\rho_2 = 1.033\,\mathrm{lg}_{10}^{-1}\frac{1}{n_2}\left(\Delta m_t - 0.1\right)$$

& $$\frac{S/\rho_1}{S/\rho_2} = \frac{D_1}{D_2}$$

$$\log\frac{D_1}{D_2} = \frac{1}{n_1}\left(\Delta m_t - 0.1\right) - \frac{1}{n_2}\left(\Delta m_t - 0.1\right)$$

$$\Delta m_t = \frac{0.1\left(\frac{1}{n_1} - \frac{1}{n_2}\right) + \log\frac{D_1}{D_2}}{\left(\frac{1}{n_1} - \frac{1}{n_2}\right)}$$

$$\Delta m_t = 0.1 + \frac{\log\frac{D_1}{D_2}}{\left(\frac{1}{n_1} - \frac{1}{n_2}\right)} = 0.1 + \log\frac{D_1}{D_2}.\frac{n_1.n_2}{\left(n_2 - n_1\right)}$$

e.g.

$$D_1 = 6" \quad n_1 = 6.5$$

$$D_2 = 2" \quad n_2 = 12$$

$$\Delta m_t = 0.1 + \log 3 \times \frac{6.5 \times 12}{\left(12 - 6.5\right)} = 6.866$$