USING a NEWTONIAN REFLECTOR for DOUBLE STAR WORK

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Introduction

To be able to resolve an equal pair at Dawes' limit, or detect the elongation in the Airy disc of a pair within Dawes' limit a Newtonian reflector needs to be collimated within a certain tolerance. This tolerance depends on three Seidel errors, coma, astigmatism and distortion, which may be grouped together to define the 'angular aberration', or AA.

Furthermore, if a separation measure of a very wide pair is to be made to an accuracy of the Airy disc diameter, the diffraction limited field of view (fov) needs to be marginally wider than the separation of the pair.

Angular aberrations

There is a direct relationship between the angular aberration blur size, the diameter of the diffraction limited field of view, and the collimation tolerance.

The angular aberrations AA of a paraboloidal mirror at prime focus (stop at surface) are[1]:-

$$AA = 3a_1 \frac{y^2}{R^2}\theta + 2a_2 \frac{y}{R}\theta^2 + a_3\theta^3$$

where for a paraboloid: $R = 2f$; $a_1 = 0.25$; $a_2 = -0.5$; $a_3 = +1$

Substituting the focal ratio: $F = f/D$ we get:

$$AA = \frac{3}{16F^2}\theta + \frac{1}{2F}\theta^2 + \theta^3$$

where $\theta = y/f$ and y is the linear ray height, and the terms in θ, θ^2 and θ^3 represent the coma, astigmatism and distortion respectively.

If an observer is going to be able to tell whether a double star below Dawes' limit is elongated, the telescope must be collimated with an accuracy such that:

$$AA \leq \frac{1.22\lambda}{D} \quad \text{where} \quad \lambda = 21.65 \text{ x } 10^{-6} \text{ ins}$$

The distance off axis where AA exceeds this limit defines both the angular collimation tolerance and the radius of the diffraction limited field of view.

Neglecting distortion because θ^3 is very small for sensible off axis distances:

$$\frac{1}{2F}\theta_{max}^2 + \frac{3}{16F^2}\theta_{max} - \frac{1.22\lambda}{D} = 0$$

$$\theta_{max} = \frac{-3}{16F} \pm \sqrt{\frac{9}{256F^2} + \frac{2.44\lambda F}{D}} \tag{1}$$

Table 1

Aperture D inches	f/ratio F	tolerance arcmin	angular fov* $2\theta_{max}$ arcsec	linear fov* 2d inches	Note
24	4	0.32	38.8	0.0180	2
18	4.5	0.54	65.3	0.0256	2
12.5	7	1.88	225.7	0.0957	1
10	6.3	1.91	228.7	0.0699	1
10.25	10.3	4.83	579.7	0.2967	4
10	12.8	7.04	887.8	0.5509	4
9.333	8	3.26	391.0	0.1415	3
8.5	7	2.75	330.4	0.0953	3
8	6	2.16	259.1	0.0603	3
6	8	2.54	305.4	0.0711	3
4	12	8.11	973.5	0.2265	3

*diffraction limited
Notes

1. Traditional mid-aperture Newtonians, easy to collimate although collimation may need checking occasionally.

2. Typical large Newtonians, almost impossible to collimate to required accuracy, and will not maintain collimation due to mirror cell flexure and tube flexure. Coma is almost certainly present on mechanical axis due to slight collimation errors. Impossibly small diffraction limited field of view.

3. Traditional small to mid-aperture Newtonians, very easy to collimate, normally maintain collimation, coma not normally present in a high power field (x25 per inch of aperture plus).

4. Newtonians specifically designed for double star work. Note the wide diffraction limited fields of view and the slack collimation tolerances. Collimation is simple to achieve and may be readily maintained.

The above list of some common apertures and focal ratios of Newtonian reflectors provides information on the collimation tolerances and diameters of diffraction limited fields of view. Note how the 'traditional' or 'classic' form of Newtonian has much less stringent collimation tolerances, and far wider diffraction limited fields of view.

When one considers how Browning[2], Horne & Thornthwaite[3], Calver[4], H.N. Irving & Son, and in the USA, Thomas Cave (Astrola) manufactured their now classic Newtonians, and then contrast them to modern commercial Dobsonian light buckets, is it any wonder that the latter consistently give an inferior image?

I cannot emphasise enough that the noticeable difference in image quality between a fast Newtonian and a classic Newtonian has little or nothing to do with the difference in the central obstruction ratio, and almost everything to do with coma. Dobsonian light buckets are plagued by coma whilst classical Newtonians are by and large free of it.

It is tempting to imagine that since resolution is inversely proportional to aperture, that building a large Newtonian will give you the edge when it comes to splitting pairs at or near Dawes' limit. However, unless one is prepared to go to the lengths of building and housing a 24-inch f/16 rather than the more typical 24-inch f/4, the potential gain is not going to materialise. The diffraction limited field of view, even assuming by some miracle it is within the collimation tolerance (max primary tilt of 0.009 inches), is less than the angular diameter of Jupiter.

The other misconception is that coma is related directly to the focal ratio, such that any f/4 Newtonian regardless of the aperture will exhibit the same degree of coma at the same angular distance off axis. In other words a 7-inch f/4 RFT would exhibit the same degree of coma as a 24-inch f/4, but the collimation tolerances are 1.27 and 0.32 arc minutes respectively. The reason for this is clear from equations (1) and (2). Coma is also accompanied by astigmatism, and the limit of θ_{max} is related to the aperture through the Airy disc formula $1.22\lambda/D$.
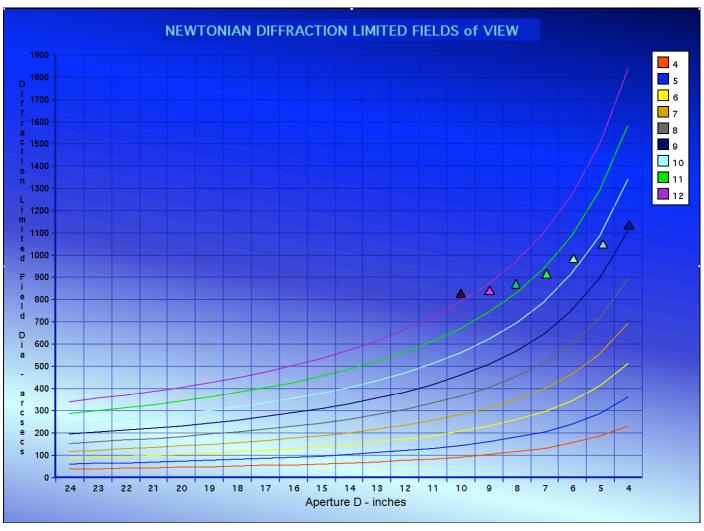
The collimation tolerance decreases in inverse proportion to the aperture. Thus for a given focal ratio, the collimation tolerance decreases in direct proportion to the increasing aperture. The same degree of coma is exhibited for any particular focal ratio not at the same angular distance off axis, but, neglecting the effects of distortion, at the same linear distance off axis. This of course is accompanied by field curvature, but because coma is far the most dominant aberration in a Newtonian, there is little to be gained by the use of a field flattener.

It is tempting therefore to employ a coma corrector, but they introduce an even worse problem as far as the double star measurer is concerned - rectilinear distortion. The image scale across the diffraction limited field begins to depart noticeably from linear, making the measurement of wide pairs problematic and their reduction awkward.

The only answer, if you want to use a Newtonian for double star work, is to build one that is as close to aplanatic as feasible. A Newtonian is only aplanatic when the focal ratio for a given aperture is such that the departure of the paraboloidal primary mirror surface from a spheroidal surface is less than $\lambda/4$. This means that even in the mid-aperture range of Newtonians the focal ratio has to be f/10 or greater, and ideally slower than f/12.

Note how my present f/10.3 setup has ample diffraction limited field of view, almost 10 arc minutes, making it an ideal system for measuring very wide pairs. My replacement f/12.8 setup has a diffraction limited field of view of almost 15 arc minutes equivalent to 0.55 inches. There is no other Newtonian in the UK with a similar high power performance specification. Note also the extraordinarily wide collimation tolerances on either of these setups, compared to any of the others. Collimation is painless and may be done using an 'EZ' laser[5] without having to then refine collimation on a bright star.

Plotting the diameter of the diffraction limited field of view against aperture for particular focal ratios produces the following result:

NEWTONIAN DIFFRACTION LIMITED FIELDS of VIEW

*The key refers to the focal ratio.

Figure 1: Newtonian diffraction limited fields of view versus Aperture

Note how the plot in Fig. 1 falls into two distinct zones. Newtonians in the darker zone have wide diffraction limited fields whilst those in the lighter zone do not. The triangular symbols plot the aplanatic limit where the departure of the paraboloidal from the spheroidal surface is $\lambda/4$.

The focal ratio for a given aperture at which the departure of the paraboloidal from the spheroidal surface is $\lambda/4$ is calculated by subtracting the sagittal depth for a parabola from the sagittal depth for a sphere and calculating back to find at which F number the difference is $\lambda/4$.[6]

Sagitta for sphere: $\sigma_s = R - \sqrt{R^2 - r^2}$

Sagitta for parabola: $\sigma_p = \dfrac{R^2}{2r}$ where r is the mirror radius

Thus when the Newtonian is aplanatic:

$$\sigma_s - \sigma_p - \frac{21.67 \times 10^{-6}}{4} = 0$$

$$R - \sqrt{R^2 - y^2} - \frac{R^2}{2y} - \frac{21.67 \times 10^{-6}}{4} = 0$$

Since $R = 2f$; $\qquad y_{max} = D/2$; $\qquad f = FD$

$$2FD - \sqrt{4F^2D^2 - \frac{D^2}{4} - \frac{21.67 \times 10^{-6}}{4}} = 0 \qquad\qquad (2)$$

Solutions to the above formula in aperture range 4 to 24 inches are tabulated below:

Aperture inches	focal ratio	Aperture inches	focal ratio	Aperture inches	focal ratio
24	16.3	17	14.5	10	12.2
23	16	16	14.2	9	11.8
22	15.8	15	13.9	8	11.3
21	15.6	14	13.6	7	10.8
20	15.3	13	13.3	6	10.3
19	15	12	12.9	5	9.7
18	14.8	11	12.6	4	9

Conclusions

The conclusion is self evident. There is a practical upper limit to the size of Newtonian that may be useful for double star work, and it lies about the 8-inch f/11 to 10-inch f/12 mark. The Newtonian design does not scale at all well, and any aplanatic Newtonian bigger than 8-inches to 10-inches grows to gargantuan proportions.

References

1   Schroeder, Daniel J., 'Astronomical Optics' Academic Press, 1987, p50 eq4.3.2

2   Browning, John, 'A Plea for Reflectors' 6th edition, 1876 trade catalogue, p29 fig.8

3   Thornthwaite, W., 'Hints on Reflecting & Refracting Telescopes' 6th edition 1895 trade catalogue, p32 fig.18

4   Calver, George, 'Hints on Silvered-Glass Reflecting Telescopes' 1877 trade catalogue p17 fig.1 & Warren Hall & Lovitt, 1880 trade catalogue p61 fig.1

5   EZ Collimator HQ invented by Chad LaFever <http://www.sunflower.com/%7Eclafever/bezt/html/ezcollimatorhq.html>

6   Rutten, Harrie and van Venrooij, Martin, 'Telescope Optics Evaluation and Design' Willmann-Bell, 1988, p46